

國立政治大學 114 學年度第 2 學期 期中考 考試命題紙

考試科目：統計學 (二)

開課班別：統計學整合開課

命題教授：吳漢銘

考試日期：4/14(二)13:10-14:50

※准帶項目打「O」，否則打「×」

1. 需加發計算紙或答案紙請在試題內封袋備註。
2. 為環保節能減碳，試題一律採雙面印刷，有特殊印製需求，請註記：**A** 卷

本試題共3頁，印刷份數：67 份

計算機	課本	筆記	字典	手機平板筆電
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備註：注意事項要看!! (ch10~12)

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注意事項:

- (1) 在答題紙上填寫學號和姓名。請在答題紙右上角標明你使用的試卷編號：A (預設) 或 B。
- (2) 所有問題可用中文或英文作答 (無需考慮語法和拼寫)。
- (3) 請按題號順序作答。總分為 120 分。
- (4) 建議使用深色原子筆 (允許使用鉛筆)。禁用手機 3C 產品。可使用計算機 (無程式功能)。
- (5) 計算過程 (針對 IV 和 V 部分) 必須寫出 (計算至小數點後 4 位)。
- (6) 答題紙與試題卷須一併交回。
- (7) 作弊學生當次及日後考試試卷將不予批改，情節嚴重將報校處理。

(-) **宣誓詞** (0%): 複寫下列宣誓詞至答案卷的第一頁最上面。(不寫扣 10 分)

0. "本人姓名 恪遵各項考試規則，若如違反，願受校方最嚴厲處罰，謹誓。"

(I) **選擇題** (30%); select one correct answer.

1. (7%) Which of the following is a key conceptual characteristic of the chi-square distribution used for making inferences about a single population variance?

(A) It is perfectly symmetric around zero, regardless of the sample size used in the analysis. (B) It can take on both positive and negative values depending on whether the sample variance is larger or smaller than the population variance. (C) Its exact shape depends strictly on the degrees of freedom, becoming less skewed and more symmetric as the degrees of freedom increase. (D) It is defined conceptually as the ratio of two independent standard normal distributions.

2. (7%) When conducting a hypothesis test to compare two population variances ($H_0 : \sigma_1^2 = \sigma_2^2$), it is standard operational practice to assign "Population 1" to the group with the larger sample variance, forcing the F test statistic ($F = \frac{s_1^2}{s_2^2}$) to have the larger variance in the numerator. What is the primary advantage of this convention?

(A) It mathematically forces the F distribution curve to become perfectly symmetric, simplifying the calculation of the exact p -value. (B) It ensures that the calculated test statistic is always greater than or equal to 1, restricting the critical rejection region entirely to the upper tail of the F distribution. (C) It eliminates the strict theoretical requirement that the two underlying populations must be normally distributed. (D) It increases the statistical power of the test by effectively swapping the numerator and denominator degrees of freedom to maximize the critical value.

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3. (8%) Two separate Chi-Square Tests of Independence are conducted on two different datasets. Dataset A has $n = 100$, and Dataset B has $n = 10,000$. Both datasets exhibit the exact same proportion of observations in each cell of their respective 3×3 contingency tables. Which of the following statements regarding the test statistics (χ_A^2 and χ_B^2) and their conclusions is conceptually correct?

- (A) χ_A^2 will equal χ_B^2 , and their corresponding p-values will be identical because the proportions are the same. (B) χ_B^2 will be larger, but the degrees of freedom will proportionally increase, resulting in identical test conclusions. (C) χ_B^2 will be 100 times larger than χ_A^2 , making the test on Dataset B much more likely to reject the null hypothesis, regardless of the practical significance of the association. (D) The p-value for Dataset B will be larger, demonstrating that larger sample sizes logically mitigate the risk of detecting spurious associations.

4. (8%) Statistically, the calculation of the test statistic for a Chi-Square Test of Homogeneity is identical to that of a Chi-Square Test of Independence. Conceptually, what is the fundamental difference in the sampling design that distinguishes the Test of Homogeneity? (A) Homogeneity requires a single random sample drawn from one overarching population, which is then cross-classified by two variables simultaneously. (B) Homogeneity involves drawing independent random samples from predetermined sub-populations and comparing the distribution of a single categorical response across them. (C) Homogeneity tests assume that the marginal totals of both the row and column variables are entirely random and unconstrained by the researcher. (D) Homogeneity requires the categorical response variable to be strictly ordinal, whereas the Test of Independence requires purely nominal variables.

(II) 填空题 (Correct spelling should be used.) (20%, 10% each)

5. Since the chi-square test indicated that not all population proportions are equal, it is reasonable for us to proceed by attempting to determine where differences among the population proportions exist. For this we will rely on a _____ procedure that can be used to conduct statistical tests between all pairs of population proportions.

6. When conducting a chi-square test, the _____ must all be 5 or more for the chi-square test to be valid.

(III) 简答题 (20%, 10% each)(write down the statement (or definition), formula if any, interpretation)

7. Explain the fundamental difference between an independent sample design and a matched sample design when comparing two population means. What is the primary statistical advantage of utilizing a matched sample design?

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8. When constructing a confidence interval for a population mean using the t -distribution or z -distribution, the interval is perfectly symmetric around the sample mean. However, a confidence interval for a population variance is not symmetric around the sample variance. Explain the statistical reason for this asymmetry (不對稱).

(IV) 計算題 (30%)

9. The following data are believed to have come from a normal distribution. Use the goodness of fit test and $\alpha = 0.05$ to test this claim. (use six classes, each with the probability of 0.1667. $\bar{x} = 22.8, s = 6.27, \alpha = 0.05$) ($P(Z \leq -0.97) = 0.1667, P(Z \leq -0.43) = 0.3334$, where $Z \sim N(0, 1)$)

17, 23, 22, 24, 19, 23, 18, 22, 20, 13, 11, 21, 18, 20, 21,
21, 18, 15, 24, 23, 23, 43, 29, 27, 26, 30, 28, 33, 23, 29

Probability Table (機率表)

Area in Upper Tail	0.20	0.10	0.05	0.025	0.01	0.005
χ^2 value ($df = 1$)	1.642	2.706	3.841	5.024	6.635	7.879
χ^2 value ($df = 2$)	3.219	4.605	5.991	7.378	9.210	10.597
χ^2 value ($df = 3$)	4.642	6.251	7.815	9.348	11.345	12.838
χ^2 value ($df = 4$)	5.989	7.779	9.488	11.143	13.277	14.860

(V) 加分題 (20%)

10. Although both tests utilize the exact same chi-square test statistic formula, what is the fundamental difference in the sampling design between a Chi-Square Test for the Equality of Population Proportions (also known as a Test of Homogeneity) and a Chi-Square Test of Independence?

注意：1、考試求公平及公正，請同學務必自律，維護學校與學生之榮譽。

2、考試時不得有交談、窺視、夾帶、抄襲、傳遞、代考或其它作弊等舞弊行為，考畢務必交卷，不得攜卷出場，違者依考場規則議處。