Statistics Quiz (2) (Nov. 25)

Answers and Computations

I. Multiple Choice (30% ; 10% each)

1. (A) 0.1606
2. (D) height of the function at x
3. (B) difference between the value of the sample mean and the value of the population mean.

II. Fill in the Blanks (20%; 10% each)

4. probability distribution
5. sample mean ($\bar{x}$) ; Central Limit Theorem(CLT)

III. Short Answer (20%; 10% each)

6. Providing decision-making information. Namely, once a particular distribution is established for a particular application, it can be used to obtain probability information about the problem.
7. **Use:** The value of the sample statistic is used to **estimate** the value of the corresponding population parameter.

   Inequality Reason: This difference is because a sample, and not a census of the entire population, is being used to develop the sample statistics.

   More complete version: The relationship is that the sample statistic (e.g., $\bar{x}$) is used as an estimate of the unknown population parameter (e.g., μ). We calculate the statistic from our sample data to infer or make a best guess about the true value of the population parameter. However, the sample statistic is not always exactly equal to the population parameter due to sampling variability (also called sampling error). This occurs because a sample is only a subset of the population, and different random samples from the same population will yield slightly different values for the statistic (like $\bar{x}$). This natural variation from sample to sample prevents any single statistic from being a perfect match to the parameter.

IV. Computation (30%; 15% each)

8. (a)

   Given $n = 175, p = 0.467$.

   Let X be the number of visitors who entered through the Beaver Meadows park entrance.

Check Normal Approximation Conditions:

$$np = 175 \times 0.467 = 81.725 \geq 5$$

$$n(1-p) = 175 \times 0.533 = 93.275 \geq 5$$

Thus, Normal Approximation is appropriate.

Hence :

$$X \sim N(\mu, \sigma^2), \quad \mu = np = 81.725, \quad \sigma^2 = np(1-p) \approx 43.56$$

Calculation:

$$\begin{aligned} P(70 \leq X < 80) &\approx P(69.5 \leq X \leq 79.5) \\ &= P\left(\frac{69.5 - 81.725}{\sqrt{43.56}} \leq Z \leq \frac{79.5 - 81.725}{\sqrt{43.56}}\right) \\ &= P(-1.85 \leq Z \leq -0.34) \\ &= P(Z \leq -0.34) - P(Z \leq -1.85) \\ &= (1 - P(Z \leq 0.34)) - (1 - P(Z \leq 1.85)) \\ &= (1 - 0.6331) - (1 - 0.9678) \\ &= \mathbf{0.3347} \end{aligned}$$

(b)

Given $n = 175, p = 0.227$.

Let Y be the number of visitors who had no recorded point of entry.

Check Normal Approximation Conditions:

$$np = 175 \times 0.227 = 39.7250 \geq 5$$

$$n(1-p) = 175 \times 0.773 = 135.2750 \geq 5$$

Thus, Normal Approximation is appropriate.

Hence:

$$Y \sim N(\mu, \sigma^2), \quad \mu = np = 39.725, \quad \sigma^2 = np(1-p) \approx 30.71$$

Calculation:

$$\begin{aligned} P(Y > 45) &\approx P(Y \geq 45.5) \\ &= P\left(Z \geq \frac{45.5 - 39.725}{\sqrt{30.71}}\right) \\ &= P(Z \geq 1.04) \\ &= 1 - P(Z < 1.04) \\ &= 1 - 0.8508 \\ &= \mathbf{0.1492} \end{aligned}$$

V. Bonus Question (+20 %)

9. If the Poisson distribution provides an appropriate description of the number of occurrences per interval, then exponential distribution provides a description of the length of the interval between occurrences.