114-1 Statistics (I) Midterm Exam Solution(Version A)

I. Multiple Choice (30%, 6% each)

1. (C)
2. (D)
3. (C)
4. (D)
5. (B)

II. Fill-in-the-blank (10%, 5% each)

6. Simpson's paradox
7. 68%, 95%, 99% (or 68.26%, 95.44%, 99.74%)

III. Short Answer (20%, 10% each)

8. Can write in Chinese
(a) Scatter Diagram and Trendline (for two quantitative variables).
(b) Side-by-Side Bar Chart (for comparison, typically at least one categorical variable).
(c) Stacked Bar Chart (for two categorical variables).

9. A Continuous Random Variable is one that can assume any numerical value within an interval or a collection of intervals.

IV. Calculation (40%, 20% each)

10. **Prostate Cancer Screening (Bayes' Rule)**
Let
C: event male patient has prostate cancer;
$+$: event positive PSA test for prostate cancer;
$-$: event negative PSA test for prostate cancer.

Given:
$P(C) = 0.02,\ P(C^c) = 0.98,$
$P(+|C) = 0.80,\ P(+|C^c) = 0.75,\ P(-|C) = 0.20.$

Bayes' Formula:

$$P(C|+) = \frac{P(C)P(+|C)}{P(C)P(+|C) + P(C^c)P(+|C^c)}$$
$$= \frac{(0.02)(0.80)}{(0.02)(0.80) + (0.98)(0.75)} = \underline{0.0213}$$

Similarly,

$$P(C|-) = \frac{P(C)P(-|C)}{P(C)P(-|C) + P(C^c)P(-|C^c)} = \frac{(0.02)(0.20)}{(0.02)(0.20) + (0.98)(0.25)} = \underline{0.0161}$$

11. **Introductory Statistics Course Withdrawals.**
Given $X \sim \text{Binomial}(n = 20, p = 0.20)$.

(a) $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2) = 0.0115 + 0.0576 + 0.1369 = \underline{0.2060}$

(b) $P(X = 4) = \underline{0.2182}$

(c) $P(X > 3) = 1 - P(X \leq 3) = 1 - [0.2060 + 0.2054] = \underline{0.5886}$

(d) $E(X) = np = 20(0.20) = \underline{4}, \quad Var(X) = np(1-p) = 20(0.20)(0.80) = \underline{3.2}$

V. Bonus (20%)

12. **Days Listed Until Sold — Are A and B Independent?**
Let

A: "Listed more than 90 days";
B: "Listed price under \$150,000.

Step 1: Calculate probabilities ($N = 800$).
$\#\{\text{A}\} = 10 + 80 + 100 + 10 = 200, \quad P(A) = 200/800 = 0.25$
$\#\{\text{B}\} = 50 + 40 + 10 = 100, \quad P(B) = 100/800 = 0.125$
$\#\{\text{A} \cap B\} = 10, \quad \text{P(A} \cap B) = 10/800 = 0.0125$

Step 2: Compare using multiplication law.
$P(A)P(B) = (0.25)(0.125) = 0.03125$

Since $P(A \cap B) = 0.0125 \neq 0.03125$, the two events are **not independent**.
(Definition: Two events A, B are independent if $P(A|B) = P(A)$ or $P(B|A) = P(B)$.)