113 - 2 - Statistics

Midterm - Answers **A**

April 28, 2025

(I) Multiple choice

1. A
2. D
3. C
4. B

(II) Fill-in-the-blank

5.

$$\frac{1}{F(0.025, 19, 20)}$$

6. Goodness of fit test and test of Homogeneity

(III) Short answer

7. The differences between the paired observations are approximately normally distributed.

8. The Level of Significance, often denoted by the Greek letter α (alpha), is the probability of rejecting the null hypothesis when it is actually true, expressed as an equality.

(IV) Calculation

9.

a. Hypotheses:

- H_0: Employment plan is independent of the type of company.
- H_a: Employment plan is not independent of the type of company.

Table 1: Observed Frequencies (f_{ij})

Employment Plan	Private	Public	Total
Add employees	37	32	69
No change	19	34	53
Lay off employees	16	42	58
Total	72	108	180

Table 2: Expected Frequencies (e_{ij})

Employment Plan	Private	Public	Total
Add employees	27.6	41.4	69
No change	21.2	31.8	53
Lay off employees	23.2	34.8	58
Total	72.0	108.0	180

Table 3: Chi-Square Components $\left(\frac{(f_{ij}-e_{ij})^2}{e_{ij}}\right)$

Employment Plan	Private	Public	Row Total
Add employees	3.20	2.13	5.33
No change	0.23	0.15	0.38
Lay off employees	2.23	1.49	3.72
		Total χ^2	**9.43**

b. Test Statistic:

$$\chi^2 = \mathbf{9.43}$$

c. Degrees of Freedom:

Degrees of freedom $= (r-1)(c-1) = (3-1)(2-1) = 2$.

Using the χ^2 table with df $= 2$, $\chi^2 = 9.43$ shows that the p-value is between 0.01 and 0.005.

Because the p-value ≤ 0.05, we reject H_0.

d. Critical Value Approach:

Using the chi-square table, we find that $\chi^2_{0.05,\,2} \approx 5.991$.

Reject the null hypothesis if $\chi^2 \geq \chi^2_{0.05,\,2}$.

Since $9.43 > 5.991$, we reject H_0.

e. Conclusion:

We conclude that the employment plan is not independent of the type of company. Thus, we expect the employment plan to differ for private and public companies.

f. Column Probabilities:
For example, $37/72 = 0.5139$.

Table 4: Column Probabilities

Employment Plan	Private	Public
Add employees	0.5139	0.2963
No change	0.2639	0.3148
Lay off employees	0.2222	0.3889

Employment opportunities look to be much better for private companies with over 50% of private companies planning to add employees (51.39%). Public companies have greater proportions of no change and lay-off employees planned. 38.89% of public companies are planning to lay off employees over the next 12 months.

Also, $69/180 = 0.3833$, or 38.33% of the companies in the survey are planning to hire and add employees during the next 12 months.

(V) Bonus

10.

In a two-tailed chi-square test, we are testing:

$$H_0 : \sigma^2 = \sigma_0^2 \quad \text{vs.} \quad H_a : \sigma^2 \neq \sigma_0^2$$

We compute the test statistic:

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

This statistic follows a chi-square distribution with $df = n - 1$ degrees of freedom under H_0.

The chi-square distribution is skewed, so you **cannot just "mirror" a value across the center** like you would with a normal distribution. In a two-tailed test, we are concerned with extreme values in both tails — very small and very large values.

Suppose John calculates a chi-square statistic and looks up its right-tail probability (e.g., $P(\chi^2 \geq \chi^2_{\text{obs}})$).

In a two-tailed test, the p-value is:

$$\text{p-value} = P(\chi^2 \leq \chi^2_L) + P(\chi^2 \geq \chi^2_U)$$

If χ^2_{obs} lies far in the upper tail, the lower tail probability $P(\chi^2 \leq \chi^2_L)$ corresponds to an equally extreme value in the lower tail.

Since the distribution is skewed, you cannot reflect across the mean. Thus, a common approximation is:

$$\text{p-value} \approx 2 \times (\text{upper tail probability of } \chi^2_{\text{obs}})$$

This gives a conservative approximation, valid even though the chi-square distribution is not symmetric.