113 - 1 - Statistics

Midterm-Answers

November 2024

(I) Multiple choice

1.A **2.**B **3.**D **4.**C

(II) Fill-in-the-blank

5.

$$\sum f_i = 2 + 4 + 6 + 4 + 2 = 18$$

$$\sum f_i m_i = 59.0 + 198.0 + 417.0 + 358.0 + 219.0 = 1251.0$$

$$Mean = \frac{\sum f_i m_i}{\sum f_i} = \frac{1251.0}{18} \approx 69.5$$

The mean is 69.5 minutes.

6.

$$P(2or3children) = P(2children) + P(3children)$$

$$P(2or3children) = 0.600 + 0.150 = 0.750$$

The probability that the couple will have either 2 or 3 children is 0.750.

(III) Short answer

7. The probability of the intersection of two events is called joint probability. Or The probability of two events both occurring.

8. Baye's theorem is a method used to compute posterior probabilities. The Bayes' theorem for the case of two events:

$$P(A_1 \mid B) = \frac{P(A_1)P(B \mid A_1)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2)}$$

$$P(A_2 \mid B) = \frac{P(A_2)P(B \mid A_2)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2)}$$

(IV) Calculation

9.

Undergraduate Major	**Business**	**Engineering**	**Other**	**Totals**
Full-Time	0.270	0.151	0.192	0.613
Part-Time	0.115	0.123	0.149	0.387
Totals	0.385	0.274	0.341	1.000

For independence, we must have that:

$$P(A)P(B) = P(A \cap B)$$

From the joint probability table above, we have:

$$P(A) = 0.613, \quad P(B) = 0.385$$

So:

$$P(A)P(B) = (0.613)(0.385) = 0.236$$

But:

$$P(A \cap B) = 0.270$$

Because:

$$P(A)P(B) \neq P(A \cap B)$$

the events A and B are not independent.

10.

$$P(S \mid B) = \frac{(0.50)(0.75)}{(0.50)(0.75) + (0.50)(0.40)} = \frac{0.375}{0.575} = 0.65$$

(V) Bonus

11.

(a) Find $P(B)$ given:

$$P(A) = 0.4, \quad P(B|A) = 0.35, \quad P(A \cup B) = 0.69$$

Using the formula for the union of two events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

First, calculate $P(A \cap B)$:

$$P(A \cap B) = P(B|A) \times P(A) = 0.35 \times 0.4 = 0.14$$

Now, use the formula for $P(A \cup B)$:

$$0.69 = 0.4 + P(B) - 0.14$$

$$P(B) = 0.69 - 0.4 + 0.14 = 0.43$$

Thus, $P(B) = 0.43$.

(b) Find $P(B|A)$ given:

$$P(A) = 0.50, \quad P(B) = 0.40, \quad P(A \cup B) = 0.88$$

Using the formula for the union of two events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

First, calculate $P(A \cap B)$:

$$0.88 = 0.50 + 0.40 - P(A \cap B)$$

$$P(A \cap B) = 0.50 + 0.40 - 0.88 = 0.02$$

Now, use the definition of conditional probability:

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.02}{0.50} = 0.04$$

Thus, $P(B|A) = 0.04$.

(c) Find $P(A_1 \cap B)$ given:

$$P(A_1) = 0.40, \quad P(A_2) = 0.60, \quad P(A_1 \cap A_2) = 0, \quad P(B|A_1) = 0.20, \quad P(B|A_2) = 0.05$$

To find $P(A_1 \cap B)$, use the definition of conditional probability:

$$P(A_1 \cap B) = P(B|A_1) \times P(A_1)$$

Substituting the known values:

$$P(A_1 \cap B) = 0.20 \times 0.40 = 0.08$$

Thus, $P(A_1 \cap B) = 0.08$.

(d) Find $P(A_1|B)$ given:

$$P(A_1) = 0.40, \quad P(A_2) = 0.60, \quad P(A_1 \cap A_2) = 0, \quad P(B|A_1) = 0.20, \quad P(B|A_2) = 0.05$$

To find $P(A_2|B)$, use Bayes' Theorem:

$$P(A_2|B) = \frac{P(A_2 \cap B)}{P(B)}$$

First, calculate $P(A_2 \cap B)$:

$$P(A_2 \cap B) = P(B|A_2) \times P(A_2) = 0.05 \times 0.60 = 0.03$$

Next, calculate $P(B)$ using the law of total probability:

$$P(B) = P(A_1 \cap B) + P(A_2 \cap B) = 0.08 + 0.03 = 0.11$$

Now, use Bayes' Theorem:

$$P(A_2|B) = \frac{0.03}{0.11} \approx 0.2727$$

Thus, $P(A_2|B) \approx 0.2727$.