微 積 分

THOMAS' CALCULUS 12/E

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開課班級： 通訊 1/電資院 1

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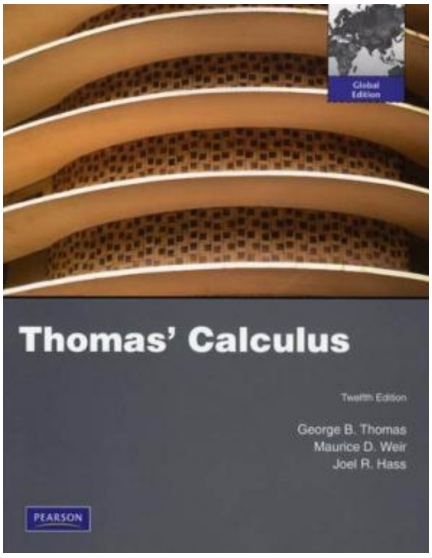

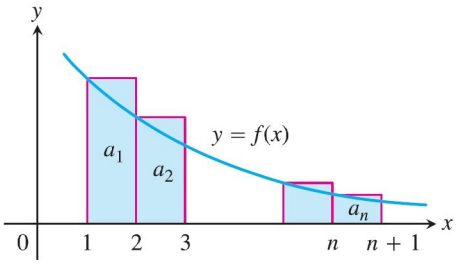

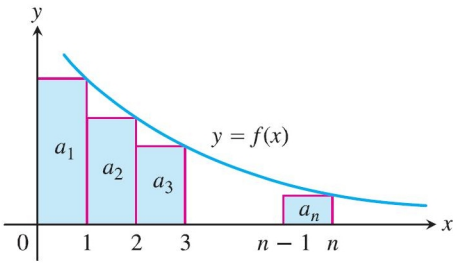

109 學年度第 2 學期

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附錄：歷年小考題、期中考題、期末考題

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叮嚀

A. 平常就要唸書算題目!

B. 考過的題目，要主動訂正。

C. 上課以「互相尊重」為最高原則並盡到「告知老師」的義務。

D. 上課可小聲討論、可上廁所安靜去回、可飲食。(但請一定要維護教室整潔)

E. 四不一要: 「上課不聊天，睡覺不趴著，手機不要滑，考試不作弊，要認真。」

THOMAS' CALCULUS (12/E)

7.1 Inverse Function and Their Derivatives

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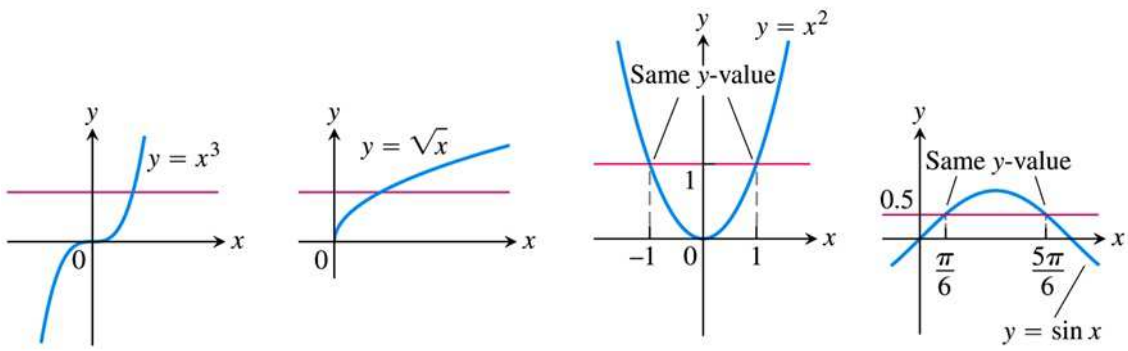
1 One-to-One Functions and Inverse Functions

1.1 *Definitions: One-to-One Function*

> A function $f(x)$ is one-to-one on a domain D if _____ whenever _____ in D.

1.2 One-to-one: (a) $y = x^3$, (b) $y = \sqrt{x}$. (圖示如下)

Not one-to-one: (c) $y = x^2$, (d) $y = \sin x$. (圖示如下)



1.3 A function $y = f(x)$ is one-to-one if and only if its graph interests each _____ at most _____.

1.4 *Definitions: Inverse Function*

> Suppose that f is a one-to-one function on a domain D with range R. The inverse function _____ is defined by
>
> _____ if _____
>
> The _____ of f^{-1} is _____ and the _____ of f^{-1} is _____.

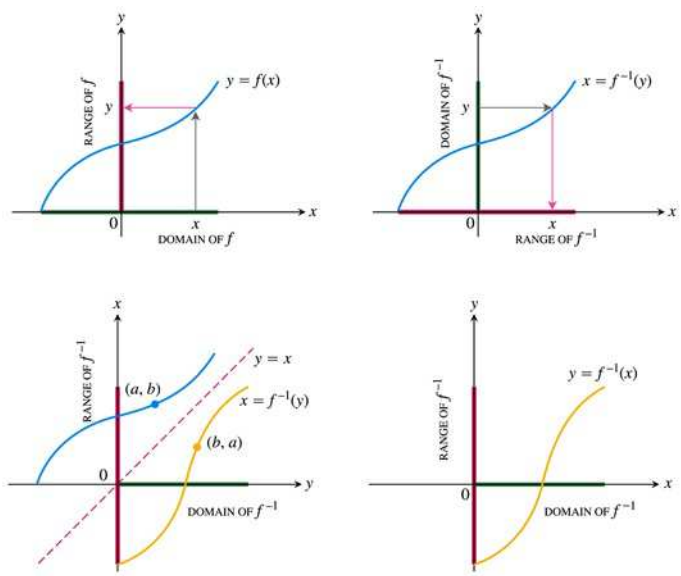
1.5 $(f^{-1} \cdot f)(x) =$ _____ for all x in the domain of f.

1.6 $(f \cdot f^{-1})(y) =$ _____ for all y in the domain of f^{-1}.

1.7 Only a one-to-one function can have an _____.

2 Finding Inverses

2.1 Determining the graph of $y = f^{-1}(x)$ from the graph of $y = f(x)$. (圖示如下)



2.2 Pass from f to f^{-1}.

(a) Solve the equation _____ for x. This gives a formula _____ where x is expressed as a function of y.

(b) Interchange _____, obtaining a formula _____ where f^{-1} is expressed in the conventional format with x as the _____ variable and y as the _____.

✐ **Ex. 1** ..... (`example3, p364`)

Find the inverse of $y = \frac{1}{2}x + 1$, expressed as a function of x.

sol:

✐ Ex. **2** ..... (`example4, p364`)

Find the inverse of the function $y = x^2,\ x \geq 0$, expressed as a function of x.

sol:

3 Derivatives of Inverses of Differentiable Functions

3.1 $f(x) = (1/2)x + 1$ and $f^{-1}(x) =$_____.

$$\frac{d}{dx}f(x) = \underline{\qquad\qquad\qquad}$$

$$\frac{d}{dx}f^{-1}(x) = \underline{\qquad\qquad\qquad}$$

3.2 *Theorem 1: The Derivative Rule for Inverses*

(a) If f has an interval I as domain and $f'(x)$ exists and is never zero on I, then f^{-1} is _____ at every point in its domain.

(b) The value of $(f^{-1})'$ at a point b in the domain of f^{-1} is the _____ of f' the value of at the point $a = f^{-1}(b)$:

$$(f^{-1})'(b) = \underline{\qquad\qquad} \quad \text{or} \quad \frac{d}{dx}f^{-1}\bigg|_{x=b} = \underline{\qquad\qquad}$$

3.3 When $y = f(x)$ is differentiable at $x = a$ and we change x by a small amount dx, the corresponding change in y is approximately _____. This means that y changes about _____ times as fast as x when $x = a$ and that x changes about _____ times as fast as y when $y = b$.

3.4 It is reasonable that the derivative of f^{-1} at b is the _____ of the derivative of f at a.

✐ Ex. **3** ..... (`example5, p366`)

Apply The Derivative Rule for Inverse Theorem to the function $f(x) = x^2$, $x \geq 0$.

sol:

✐ Ex. 4 ..... (`example6, p366`)

Let $f(x) = x^3 - 2$. Find the value of df^{-1}/dx at $x = 6 = f(2)$ without finding a formula for $f^{-1}(x)$.

sol:

實習課練習 (EXERCISE 7.1)

21. Let $f(x) = x^3 - 1$. Find a formula for f^{-1}.

22. Let $f(x) = x^2 - 2x + 1,\ x \geq 1$. Find a formula for f^{-1}.

33. Let $f(x) = x^2 - 2x,\ x \leq 1$. Find f^{-1} and identify the domain and range of f^{-1}.

37. Let $f(x) = 5 - 4x,\ a = 1/2$. Find $f^{-1}(x)$. Evaluate df/dx at $x = a$ and df^{-1}/dx at $x = f(a)$.

38. Let $f(x) = 2x^2,\ x \geq 0,\ a = 5$. Find $f^{-1}(x)$. Evaluate df/dx at $x = a$ and df^{-1}/dx at $x = f(a)$.

41. Let $f(x) = x^3 - 3x^2 - 1,\ x \geq 2$. Find the value of df^{-1}/dx at the point $x = -1 = f(3)$.

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7.2 Natural Logarithms

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1 Definition of the Natural Logarithm Function

1.1 The natural logarithm of a positive number x, written as _____, is the value of an integral.

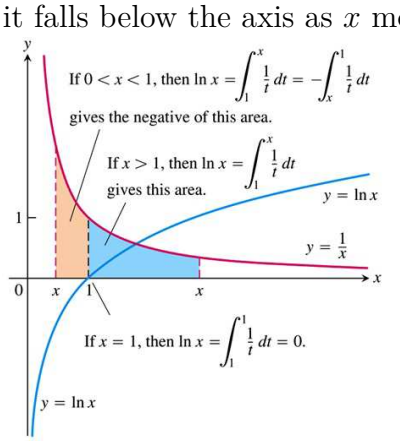
1.2 *Definitions: The Natural Logarithm Function*

$$\ln x = \underline{\qquad\qquad\qquad}, \quad x > 0$$

1.3 If $x > 1$, then $\ln x$ is the area under the curve _____ from _____ to _____.

1.4 For $0 < x < 1$, $\ln x$ gives the _____ under the curve from _____ to _____.

1.5 The graph of $y = \ln x$ and its relation to the function $y = 1/x,\ x > 0$.
The graph of the logarithm rises above the x-axis as x moves from 1 to the right, and it falls below the axis as x moves from 1 to the left. (圖示如下)



1.6 *Definitions: The Number e*

> The number e is that number in the domain of the natural logarithm satisfying _____.

1.7 $\ln 1 =$ _____.

1.8 Geometrically, the number e corresponds to the point on the x-axis for which the area under the graph of _____ and above the interval _____ is the exact area of the unit square.

2 The Derivative of $y = \ln x$

2.1 $\dfrac{d}{dx}\ln x =$ _____ $=$ _____

2.2 $y = \ln u$

$$\frac{d}{dx}\ln u = \underline{\qquad\qquad} = \underline{\qquad\qquad}, \quad u > 0$$

✐ Ex. 1 ..... (`example1, p371`)

1. $\dfrac{d}{dx}\ln 2x =$
2. $\dfrac{d}{dx}\ln(x^2+3) =$
3. $\dfrac{d}{dx}\ln ax =$

3 Properties of Logarithms

3.1 *Theorem 2: Properties of Logarithms*

> For any numbers $a > 0$ and $x > 0$, the natural logarithm satisfies the following rules:
>
> (a) Product Rule: _____
>
> (b) Quotient Rule: _____
>
> (c) Reciprocal Rule: _____
>
> (d) Power Rule: _____

✐ Ex. **2** ..... (`example2, p372`)

1. $\ln 4 + \ln \sin x =$
2. $\ln \dfrac{x+1}{2x-3} =$
3. $\ln \sec x =$
4. $\ln \sqrt[3]{x+1} =$

4 The Integral $\int (1/u) du$

4.1 If u is a differentiable function that is never zero,

$$\int \frac{1}{u}\, du = \underline{\qquad\qquad}.$$

4.2 If $u = f(x)$, then $du =$ _____ and

$$\int \frac{1}{u}\, du = \underline{\qquad\qquad\qquad} = \underline{\qquad\qquad\qquad}.$$

✐ Ex. **3** ..... (`example3, p374`)

$$\int_0^2 \frac{2x}{x^2-5}\, dx$$

sol:

5 The Integral of $\tan x$, $\cot x$, $\sec x$ and $\csc x$

5.1 $\displaystyle\int \tan u\, du =$ _____

Proof:

5.2 $\int \cot u \; du =$ _____

Proof:

5.3 $\int \sec u \; du =$ _____

Proof:

5.4 $\int \csc u \; du =$ _____

Proof:

✐ Ex. **4** ..... (`example4, p375`)

$$\int_0^{\pi/6} \tan 2x \; dx$$

sol:

6 Logarithmic Differentiation

✐ Ex. **5** ..... (`example5, p375`)

Find dy/dx if $y = \dfrac{(x^2+1)(x+3)^{1/2}}{x-1}, \quad x > 1.$

sol:

實習課練習 (EXERCISE 7.2)

2. Express the following logarithms in terms of $\ln 5$ and $\ln 7$. (a) $\ln(1/125)$, (b) $\ln 9.8$, (c) $\ln 7\sqrt{7}$, (d) $\ln 1225$, (e) $\ln 0.056$, (f) $(\ln 35 + \ln(1/7))/(\ln 25)$.

3. Simplify the expressions: (a) $\ln \sin\theta - \ln(\frac{\sin\theta}{5})$, (b) $\ln(3x^2 - 9x) + \ln(\frac{1}{3x})$, (c) $\frac{1}{2}\ln(4t^4) - \ln 2$.

☐ Find the derivative of y with respect to x, or t, as appropriate.

8. $y = \ln(t^3/2)$

15. $y = t(\ln t)^2$

22. $y = \dfrac{x \ln x}{1 + \ln x}$

35. $y = \displaystyle\int_{x^2/2}^{x^2} \ln\sqrt{t}\ dt$

☐ Evaluate the integrals:

38. $\displaystyle\int_{-1}^{0} \frac{3}{3x-2}\ dx$

44. $\displaystyle\int_{2}^{4} \frac{dx}{x \ln x}$

45. $\displaystyle\int_{2}^{4} \frac{dx}{x(\ln x)^2}$

52. $\displaystyle\int_{0}^{\pi/12} 6\tan 3x\ dx$

53. $\displaystyle\int \frac{dx}{2\sqrt{x} + 2x}$

☐ Use logarithmic differentiaton to find the derivative of y with respect to the given independent variable.

55. $y = \sqrt{x(x+1)}$

62. $y = \dfrac{1}{t(t+1)(t+2)}$

67. $y = \sqrt[3]{\dfrac{x(x-2)}{x^2+1}}$

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7.3 Exponential Function

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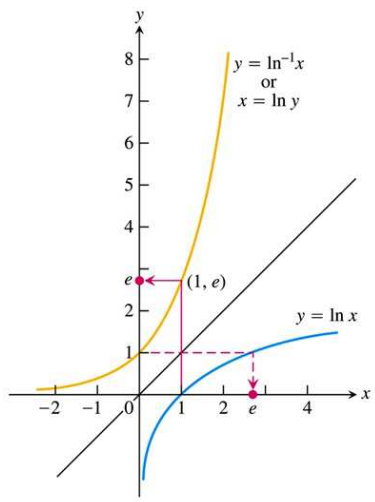
1 The Inverse of $\ln x$ and e^x

1.1 The function $\ln x$, being an increasing function of x with domain $(0, \infty)$ and range $(-\infty, \infty)$, has an inverse $\ln^{-1} x$ with domain $(-\infty, \infty)$ and range $(0, \infty)$. (圖示如下)



1.2 $\ln(e) =$_____, $e =$ _____ $=$ _____.

1.3 $e \doteq$ _____.

1.4 *Definitions: The Natural Exponential Function*

For every real number x, $e^x =$ _____ $=$ _____.

1.5 $\ln e^r =$_____ $\Rightarrow$ $e^r =$_____.

1.6 Inverse Equation for e^x and $\ln x$

$$e^{lnx} = _____, \quad \forall x > 0$$

$$\ln(e^x) = _____, \quad \forall x$$

✐ Ex. **1** ..... (`example1, p378`)

Solve the equation $e^{2x-6} = 4$ for x.

sol:

2 The Derivative and Integral of e^x

2.1 Let $f(x) = \ln x$ and $y = e^x =$ _____. Then $\dfrac{dy}{dx} =$ _____

2.2 If u is any differentiable function of x, then $\dfrac{d}{dx}\, e^u =$ _____

2.3 $\displaystyle\int e^u \, du =$ _____

✐ Ex. **2** ..... (`example2, p379`)

(a) $\dfrac{d}{dx}\,(5e^x) =$

(b) $\dfrac{d}{dx}\, e^{-x} =$

(c) $\dfrac{d}{dx}\, e^{\sin x} =$

(d) $\dfrac{d}{dx}\,(e^{\sqrt{3x+1}}) =$

✐ Ex. **3** ..... (`example3, p379`)

(a) $\displaystyle\int_0^{\ln 2} e^{3x}\, dx =$

(b) $\displaystyle\int_0^{\pi/2} e^{\sin x} \cos x\, dx =$

3 Laws of Exponents

3.1 *Definitions: General Exponential Functions*

For any numbers $a > 0$ and x, the exponential function with base a is _____.

3.2 *Theorem 3: Laws of Exponents for e^x*

For all numbers x, x_1 and x_2, the natural exponential e^x obeys the following laws:

(a) $e^{x_1} \cdot e^{x_2} =$_____

(b) $e^{-1} =$_____

(c) $e^{x_1}/e^{x_2} =$_____

(d) $((e^{x_1})^{x_2}) =$_____

Proof of Law (a):

✐ Ex. **4** ..... (`example4, p382`) Differentiate $f(x) = x^x,\ x > 0$.

sol:

4 The Number e Expressed as a Limit

4.1 *Theorem 4: The Number e as a Limit*

> The number e can be calculated as the limit: $e =$ _____.

Proof:

4.2 *Power Rule (General Form)*

> If u is a positive differentiable function of x and n is any real number, then u^n is a differentiable function of x and $\frac{d}{dx}u^n =$ _____.

4.3 Examples:

(a) $\frac{d}{dx}x^{\sqrt{2}} =$

(b) $(2 + \sin 3x)^{\pi} =$

5 The Derivative of a^u

5.1 If $a > 0$, then $\frac{d}{dx}a^x =$_____.

Proof:

5.2 If $a > 0$ and u is a differentiable function of x, then a^u is a differentiable function of x and $\frac{d}{dx}a^u =$ _____.

5.3 If $a \neq 1$, $\int a^u \, du =$ _____.

Proof:

✐ Ex. **5** ..... (`example5, p383`)

(a) $\dfrac{d}{dx} 3^x =$

(b) $\dfrac{d}{dx} 3^{-x} =$

(c) $\dfrac{d}{dx} 3^{\sin x} =$

(d) $\displaystyle\int 2^x \, dx =$

(e) $\displaystyle\int 2^{\sin x} \cos x \, dx =$

6 Logarithms with Base a

6.1 *Definitions:* $\log_a x$

For any positive number $a \neq 1$, _____ is the inverse function of a^x.

6.2 Inverse Equations for a^x and $log_a x$

$a^{\log_a x} =$_____, $\quad x > 0$

$\log_a(a^x) =$_____,$\forall x$

6.3 $\log_a x =$ _____

Proof:

6.4 Derivatives and Integrals Involving $\log_a x$:

$$\frac{d}{dx}(\log_a u) = \underline{\qquad\qquad}$$

✎ Ex. **6** ..... (`example6, p385`)

(a) $\dfrac{d}{dx}\log_{10}(3x+1) =$

(b) $\displaystyle\int \frac{\log_2 x}{x}\, dx =$

實習課練習 (EXERCISE 7.3)

☐ Solve for t.

2. (a) $e^{-0.01t} = 1000$, (b) $e^{kt} = \frac{1}{10}$, (c) $e^{(\ln 2)x} = 1/2$.

4. $e^{x^2} e^{2x+1} = e^t$

☐ Find the derivative of y with respect to x, t or θ, as appropriate.

9. $y = xe^x - e^x$

14. $y = \ln(3\theta e^{-\theta})$

20. $y = \ln(\dfrac{\sqrt{\theta}}{1+\sqrt{\theta}})$

24. $y = \displaystyle\int_{e^4\sqrt{x}}^{e^{2x}} \ln t \; dt$

25. $\ln y = e^y \sin x$

28. $\tan y = e^x + \ln x$

☐ Evaluate the integrals.

33. $\displaystyle\int 8e^{x+1} \; dx$

41. $\displaystyle\int \frac{e^{1/x}}{x^2} \; dx$

43. $\displaystyle\int_0^{\pi/4} (1 + e^{\tan\theta}) \sec^2\theta \; d\theta$

49. $\displaystyle\int \frac{e^r}{1+e^r} \; dr$

50. $\displaystyle\int \frac{dx}{1+e^x}$

☐ Find the derivative of y with respect to the given independent variable.

57. $y = 5^{\sqrt{s}}$

72. $y = \log_3 r \cdot \log_9 r$

81. $y = \log_2(8t^{\ln 2})$

☐ Evaluate the integrals.

85. $\displaystyle\int_0^1 2^{-\theta}\, d\theta$

91. $\displaystyle\int_2^4 x^{2x}(1+\ln x)\, dx$

96. $\displaystyle\int_1^e x^{(\ln 2-1)}\, dx$

99. $\displaystyle\int_1^4 \frac{\ln 2 \log_2 x}{x}\, dx$

106. $\displaystyle\int \frac{dx}{x(\log_8 x)^2}$

107. $\displaystyle\int_1^{\ln x} \frac{1}{t}\, dt, \quad x > 1$

☐ Find the derivative of y with respect to the given independent variable.

111. $y = x + 1^x$

116. $y = x^{\sin x}$

118. $y = (\ln x)^{\ln x}$

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7.5 Indeterminate Forms and L'Hopital's Rule

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1 Indeterminate Form 0/0

1.1 If the continuous functions $f(x)$ and $g(x)$ are both _____ at $x = a$, then _____ cannot be found by substituting $x = a$.

1.2 The substitution produces _____, a meaningless expression, which we cannot evaluate. We use _____ as a notation for an expression known as an _____.

1.3 *Theorem: L'Hopital's Rule (First Form)*

> Suppose that _____, that $f'(a)$ and $g'(a)$ exist, and that $g'(a) \neq 0$. Then
> $$\lim_{x \to a} \frac{f(x)}{g(x)} = \underline{\qquad\qquad}.$$

Proof:

1.4 *Theorem: L'Hopital's Rule (Stronger Form)*

> Suppose that _____, that f and g are differentiable on an open interval I containing a, and that $g'(x) \neq 0$ on I if $x \neq a$. Then
> $$\lim_{x \to a} \frac{f(x)}{g(x)} = \underline{\qquad\qquad},$$
> assuming that the limit on the right side exists.

1.5 *Using L'Hopital's Rule*

To find $\lim_{x\to a} \frac{f(x)}{g(x)}$ by L'Hopital's Rule,

(a) continue to differentiate f and g, so long as we still get the form _____ at $x = a$.

(b) But as soon as one or the other of these derivatives is different from _____ at $x = a$ we stop differentiating.

(c) L'Hopital's Rule does not apply when either the _____ or _____ has a finite _____ limit.

Ex. **1** ..... (`example1, p397`)

(a) $\lim_{x\to 0} \frac{3x - \sin x}{x} =$

(b) $\lim_{x\to 0} \frac{\sqrt{1+x} - 1}{x} =$

(c) $\lim_{x\to 0} \frac{\sqrt{1+x} - 1 - x/2}{x^2} =$

(d) $\lim_{x\to 0} \frac{x - \sin x}{x^3} =$

✐ Ex. **2** ..... (`example2, p398`)

Find $\lim_{x\to 0} \dfrac{1-\cos x}{x+x^2}$.

sol:

✐ Ex. **3** ..... (`example3, p398`)

(a) $\lim_{x\to 0^+} \dfrac{\sin x}{x^2} =$

(b) $\lim_{x\to 0^-} \dfrac{\sin x}{x^2} =$

2 Indeterminate Form $\infty/\infty, \infty\cdot 0, \infty-\infty$

2.1 L'Hopital's Rule applies to the indeterminate form _____.

2.2 If _____ and _____ as $x \to a$ then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \underline{\qquad\qquad}$$

provided the limit on the right exists.

2.3 In the notation $x \to a$ may be either _____ or _____.

2.4 Moreover $x \to a$ may be replaced by the one-sided limits _____ or _____.

✐ Ex. **4** ..... (`example4, p398`)

(a) $\lim_{x\to \pi/2} \dfrac{\sec x}{1+\tan x} =$

(b) $\lim_{x\to \infty} \dfrac{\ln x}{2\sqrt{x}} =$

(c) $\lim_{x\to\infty} \frac{e^x}{x^2} =$

✐ Ex. **5** ..... (`example5, p399`)

(a) Find $\lim_{x\to\infty}(x\sin\frac{1}{x})$.

(b) Find $\lim_{x\to 0^+}(\sqrt{x}\ln x)$.

sol:

✐ Ex. **6** ..... (`example6, p399`)

Find $\lim_{x\to 0}(\frac{1}{\sin x} - \frac{1}{x})$.

sol:

3 Indeterminate Powers

3.1 If $\lim_{x\to a} \ln f(x) = L$, then $\lim_{x\to a} f(x) =$ _____. Here a may be either finite or infinite.

3.2 *Theorem: Cauchy's Mean Value Theorem*

Suppose functions f and g are continuous on $[a, b]$ and differentiable throughout (a, b) and also suppose $g'(x) \neq 0$ throughout (a, b). Then there exists a number c in (a, b) at which

$$\frac{f'(c)}{g'(c)} = \underline{\qquad\qquad}.$$

Ex. 7 ..... (`example7, p400`)

Apply l'Hôpital's Rule to show that $\lim_{x\to 0^+}(1+x)^{1/x} = e$

sol:

Ex. 8 ..... (`example8, p400`)

Find $\lim\limits_{x\to\infty} x^{1/x}$.

sol:

實習課練習 (EXERCISE 7.5)

Use L'Hôpital Rule to find the limits.

5. $\lim_{x\to 0} \frac{1-\cos x}{x^2}$.

8. $\lim_{x\to -5} \frac{x^2-25}{x+5}$.

14. $\lim_{t\to 0} \frac{\sin 5t}{2t}$.

20. $\lim_{x\to 1} \frac{x-1}{\ln x - \sin \pi x}$.

27. $\lim_{\theta\to 0} \frac{3^{\sin\theta}-1}{\theta}$.

29. $\lim_{x\to 0} \frac{x2^x}{2^x-1}$.

34. $\lim_{x\to 0^+} \frac{\ln(e^x-1)}{\ln x}$.

41. $\lim_{x\to 1^+} (\frac{1}{x-1} - \frac{1}{\ln x})$.

46. $\lim_{x\to\infty} x^2 e^{-x}$.

48. $\lim_{x\to 0} \frac{(e^x-1)^2}{x\sin x}$.

53. $\lim_{x\to\infty} (\ln x)^{1/x}$.

58. $\lim_{x\to 0} (e^x+x)^{1/x}$.

59. $\lim_{x\to 0^+} x^x$.

60. $\lim_{x\to 0^+} (1+\frac{1}{x})^x$.

62. $\lim_{x\to\infty} (\frac{x^2+1}{x+2})^{1/x}$.

73. $\lim_{x\to\infty} \frac{e^{x^2}}{xe^x}$.

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7.6 Inverse Trigonometric Functions

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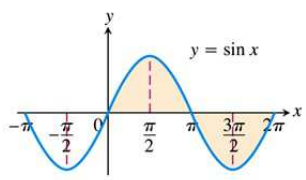
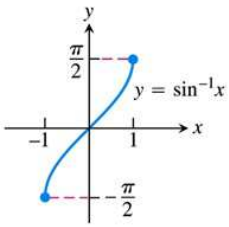
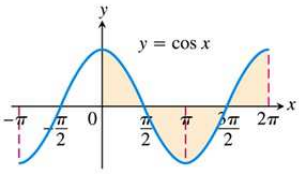
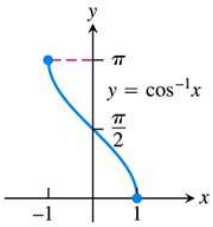
1 Defining the Inverses

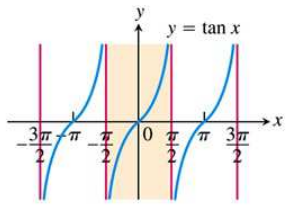
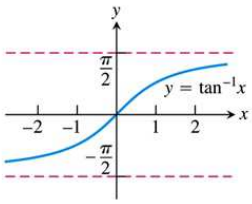
1.1 Domain restrictions that make the trigonometric functions one-to-one.

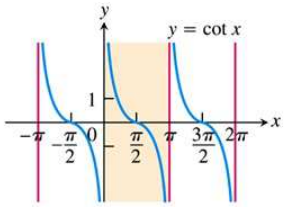
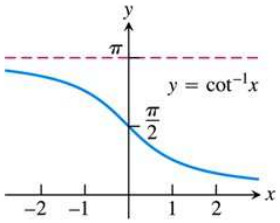
(a) $y = \sin^{-1} x$ is the number in _____ for which _____.

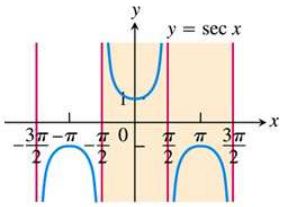
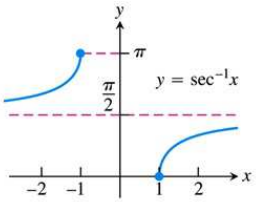
(b) $y = \cos^{-1} x$ is the number in _____ for which _____.

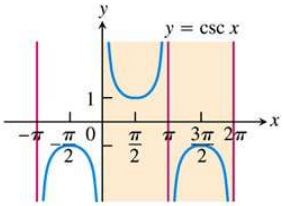
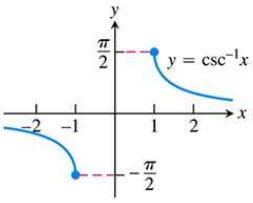
$\sin x$	$\sin^{-1} x$ or $\arcsin x$
D: _____ R: _____ 	D: _____ R: _____ 

$\cos x$	$\cos^{-1} x$ or $\arccos x$
D: _____ R: _____ 	D: _____ R: _____ 

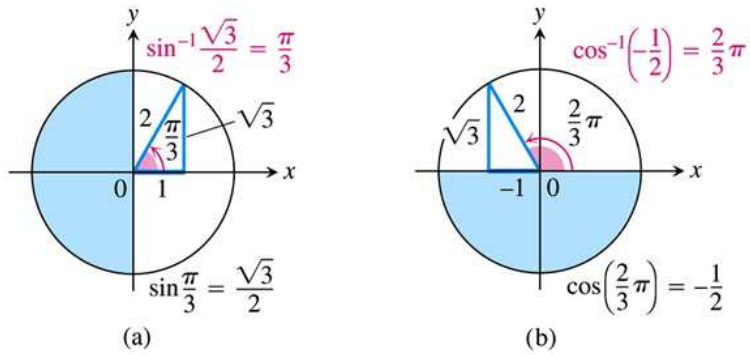
$\tan x$	$\tan^{-1} x$ or $\arctan x$
D: _____ R: _____ 	D: _____ R: _____ 

$\cot x$	$\cot^{-1} x$ or $\operatorname{arccot} x$
D: _____ R: _____ 	D: _____ R: _____ 

$\sec x$	$\sec^{-1} x$ or $\operatorname{arcsec} x$
D: _____ R: _____ 	D: _____ R: _____ 

$\csc x$	$\csc^{-1} x$ or $\operatorname{arccsc} x$
D: _____ R: _____ 	D: _____ R: _____ 

1.2 Common values of $\sin^{-1} x$:



x	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	$-1/2$	$-\sqrt{2}/2$	$-\sqrt{3}/2$
$\sin^{-1} x$						
$\cos^{-1} x$						

TABLE 1.3 Values of $\sin\theta$, $\cos\theta$, and $\tan\theta$ for selected values of θ

Degrees	**−180**	**−135**	**−90**	**−45**	**0**	**30**	**45**	**60**	**90**	**120**	**135**	**150**	**180**	**270**	**360**
θ (radians)	$-\pi$	$\frac{-3\pi}{4}$	$\frac{-\pi}{2}$	$\frac{-\pi}{4}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
$\sin\theta$	0	$\frac{-\sqrt{2}}{2}$	-1	$\frac{-\sqrt{2}}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\cos\theta$	-1	$\frac{-\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-\sqrt{3}}{2}$	-1	0	1
$\tan\theta$	0	1		-1	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$		$-\sqrt{3}$	-1	$\frac{-\sqrt{3}}{3}$	0		0

2 The Drivative of Inverse Trigonometric Functions

2.1 $(f^{-1}(x))' =$ _____ .

2.2 Find the derivative of $y = \sin^{-1} x$.

2.3 Find the derivative of $y = \tan^{-1} x$.

2.4 Find the derivative of $y = \sec^{-1} x$.

2.5 $u = u(x)$

(a) $\dfrac{d}{dx}(\sin^{-1} u) =$ _____ , $|u| < 1$.

(b) $\dfrac{d}{dx}(\tan^{-1} u) =$ _____ .

(c) $\dfrac{d}{dx}(\sec^{-1} u) =$ _____ .

2.6 *Inverse Function-Inverse Cofunction Identities*

(a) $\cos^{-1} x =$ _____ $\Rightarrow \dfrac{d}{dx}(\cos^{-1} u) =$ _____ , $|u| < 1$

(b) $\cot^{-1} x =$ _____ $\Rightarrow \dfrac{d}{dx}(\cot^{-1} u) =$ _____

(c) $\csc^{-1} x =$ _____ $\Rightarrow \dfrac{d}{dx}(\csc^{-1} u) =$ _____ , $|u| > 1$

✐ Ex. **1** ..... (`example4, p408`)

$\dfrac{d}{dx}(\sin^{-1} x^2) =$

sol:

✐ Ex. **2** ..... (`example5, p410`)

$\dfrac{d}{dx}(\sec^{-1} 5x^4) =$

sol:

3 Integration Formulas

3.1 $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} =$ _____ , $u^2 < a^2$

3.2 $\displaystyle\int \frac{du}{a^2+u^2} =$ _____

3.3 $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} =$ _____ , $|u| > a > 0$

✐ Ex. **3** ..... (`example6, p412`)

(a) $\displaystyle\int_{\sqrt{2}/2}^{\sqrt{3}/2} \frac{dx}{\sqrt{1-x^2}} =$

(b) $\displaystyle\int \frac{dx}{\sqrt{3-4x^2}} =$

(c) $\displaystyle\int \frac{dx}{\sqrt{e^{2x}-6}} =$

✐ Ex. **4** ..... (`example7(a), p412`)

Evaluate $\displaystyle\int \frac{dx}{\sqrt{4x - x^2}}$

sol:

✐ Ex. **5** ..... (`example7(b), p412`)

Evaluate $\displaystyle\int \frac{dx}{4x^2 + 4x + 2}$

sol:

實習課練習 (EXERCISE 7.6)

☐ Find the values.

9. $\sin(\cos^{-1}(\frac{\sqrt{2}}{2}))$

12. $\cot(\sin^{-1}(-\frac{\sqrt{3}}{2}))$

☐ Find the derivative of y with respect to the appropriate variable.

22. $y = \cos^{-1}(1/x)$ $y = \sin^{-1}(3/t^2)$

33. $y = \ln(\tan^{-1} x)$

41. $y = x\sin^{-1} x + \sqrt{1-x^2}$

42. $y = \ln(x^2+4) - x\tan^{-1}(\frac{x}{2})$

☐ Evaluate the integrals.

44. $\displaystyle\int \frac{dx}{\sqrt{1-4x^2}}$

46. $\displaystyle\int \frac{dx}{9+3x^2}$

47. $\displaystyle\int \frac{dx}{x\sqrt{25x^2-2}}$

63. $\displaystyle\int_0^{\ln\sqrt{3}} \frac{e^x\ dx}{1+e^{2x}}$

68. $\displaystyle\int \frac{dx}{\sqrt{2x-x^2}}$

72. $\displaystyle\int \frac{dy}{y^2+6y+10}$

79. $\displaystyle\int \frac{dx}{(x+1)\sqrt{x^2+2x}}$

83. $\displaystyle\int \frac{(\sin^{-1} x)^2\ dx}{\sqrt{1-x^2}}$

86. $\displaystyle\int \frac{dy}{(\sin^{-1} y)\sqrt{1-y^2}}$

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8.1 Integration by Parts

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1 Basic Integration Formulas

1.1 Substitution Rule: $u = g(x)$, _____

1.2 Basic formulas:

(1) $\int du =$ _____

(2) $\int k\ du =$ _____

(3) $\int (du + dv) =$ _____

(4) $\int u^n\ du =$ _____ , $n \neq -1$

(5) $\int \frac{du}{u} =$ _____

(6) $\int \sin u\ du =$ _____

(7) $\int \sec^2 u\ du =$ _____

(8) $\int \csc^2 u\ du =$ _____

(9) $\int \sec u \tan u\ du =$ _____

(10) $\int \csc u \cot u\ du =$ _____

(11) $\int \tan u\ du =$ _____ = _____

(12) $\int \cot u \, du =$ _____ $=$ _____

(13) $\int e^u \, du =$ _____

(14) $\int a^u \, du =$ _____

(15) $\int \frac{du}{\sqrt{a^2 - u^2}} =$ _____

(16) $\int \frac{du}{a^2 + u^2} =$ _____

(17) $\int \frac{du}{u\sqrt{u^2 - a^2}} =$ _____

2 Product Rule in Integral Form

2.1 Integration by parts: if f and g are differentiable functions of x,

(a) The Product Rule: $\frac{d}{dx}[f(x)g(x)] =$ _____.

(b) In term of indefinite integrals:

$$\int \frac{d}{dx}[f(x)g(x)] \, dx = \underline{\hspace{6cm}}.$$

(c) Integration by parts:

_____.

2.2 Let $u = f(x)$ and $v = g(x)$, then $du = f'(x) \, dx$ and $dv = g'(x) \, dx$. Using substitution rule, the integration by parts formula becomes _____.

2.3 Note:

(a) The goal of integration by parts is to go from an integral _____ that we don't see how to evaluate to an integral _____ that we can evaluate.

(b) The integration by parts does not always work.

2.4 Integration by Parts Formula for Definite Integrals

$$\int_a^b f(x)g'(x) \, dx = \underline{\hspace{6cm}}$$

✐ Ex. **1** ..... (`example1, p437`)

Find $\int x \cos x \; dx$

sol:

✐ Ex. **2** ..... (`example2, p3`)563

Find $\int \ln x \; dx$

sol:

✐ Ex. **3** ..... (`example3, p437`)

Find $\int x^2 e^x \; dx$

sol:

✐ Ex. **4** ..... (`example4, p438`)

Find $\int e^x \cos x \; dx$

sol:

3 Tabular Integration

✐ Ex. **5** ..... (`example7, p440`)

Evaluate $\int x^2 e^x \; dx$.

sol:

✐ Ex. **6** ..... (`example8, p441`)

Evaluate $\int x^3 \sin x \; dx$

sol:

實習課練習 (EXERCISE 8.1)

1. $\displaystyle\int x \sin \frac{x}{2}\ dx$

6. $\displaystyle\int_1^e x^3 \ln x\ dx$

16. $\displaystyle\int p^4 e^{-p}\ dp$

19. $\displaystyle\int x^5 e^x\ dx$

21. $\displaystyle\int e^\theta \sin\theta\ d\theta$

25. $\displaystyle\int e^{\sqrt{3s+9}}\ ds$

28. $\displaystyle\int \ln(x+x^2)\ dx$

36. $\displaystyle\int \frac{(\ln x)^3}{x}\ dx$

44. $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\ dx$

46. $\displaystyle\int \sqrt{x} e^{\sqrt{x}}\ dx$

62. Establish the reduction formula: $\displaystyle\int x^n \sin x\ dx = -x^n \cos x + n \int x^{n-1} \cos x\ dx.$

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8.2 Trigonometric Integrals

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1 Products of Powers of Sines and Cosines

1.1 $\int \sin^m x \cos^n x \, dx$

Case 1. If m is _____ $\Rightarrow m =$ _____, use _____

$$\sin^m x = _____ = _____ = _____$$

Case 2. If m is _____ and n is _____. $\Rightarrow n =$ _____, use _____

$$\cos^n x = _____ = _____ = _____$$

Case 3. If both m and n are _____ $\Rightarrow$ use $\sin^2 x =$ _____, $\cos^2 x =$ _____.

✐ Ex. 1 ..... (example1, p444)

Evaluate $\int \sin^3 x \cos^2 x \, dx$.

sol:

✐ Ex. **2** ..... (`example2, p445`)

Evaluate $\int \cos^5 x \; dx$.

sol:

✐ Ex. **3** ..... (`example3, p445`)

Evaluate $\int \sin^2 x \cos^4 x \; dx$.

sol:

2 Eliminating Square Roots

2.1 Use $\cos^2 \theta =$ _____

✐ Ex. **4** ..... (`example4, p446`)

Evaluate $\int_0^{\pi/4} \sqrt{1 + \cos 4x} \; dx$.

sol:

3 Integrals of Powers of $\tan x$ and $\sec x$

3.1 Use $\tan^2 x =$ _____

3.2 Use $\sec^2 x =$ _____

✐ Ex. **5** ..... (`example5, p446`)

Evaluate $\int \tan^4 x \, dx$.

sol:

✐ Ex. **6** ..... (`example6, p447`)

Evaluate $\int \sec^3 x \, dx$.

sol:

4 Products of Sine and Cosines

4.1 $\sin mx \sin nx =$ _____ .

4.2 $\sin mx \cos nx =$ _____ .

4.3 $\cos mx \cos nx =$ _____ .

✐ Ex. **7** ..... (`example7, p448`)

Evaluate $\int \sin 3x \cos 5x \; dx$.

sol:

實習課練習 (EXERCISE 8.2)

8. $\displaystyle\int_0^{\pi} \sin^5 \frac{x}{2}\ dx$

16. $\displaystyle\int 7\cos^7 t\ dt$

17. $\displaystyle\int_0^{\pi} 8\sin^4 x\ dx$

22. $\displaystyle\int_0^{\pi/2} \sin^2 2\theta \cos^3 2\theta\ d\theta$

23. $\displaystyle\int_0^{2\pi} \sqrt{\frac{1-\cos x}{2}}\ dx$

25. $\displaystyle\int_0^{\pi} \sqrt{1-\sin^2 t}\ dt$

31. $\displaystyle\int_0^{\pi/2} \theta\sqrt{1-\cos 2\theta}\ d\theta$

35. $\displaystyle\int \sec^3 x \tan x\ dx$

38. $\displaystyle\int \sec^4 x \tan^2 x\ dx$

53. $\displaystyle\int_{-\pi}^{\pi} \sin 3x \sin 3x\ dx$

55. $\displaystyle\int \cos 3x \cos 4x\ dx$

57. $\displaystyle\int \sin^2 \theta \cos 3\theta\ d\theta$

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8.3 Trigonometric Substitutions

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1 Three Basic Substitutions

1.1 Integrals involving $\sqrt{a^2 - x^2}$:

Let $x =$ _____,

$a^2 - x^2 =$ _____ $=$ _____ $=$ _____

1.2 Integrals involving $\sqrt{a^2 + x^2}$:

Let $x =$ _____,

$a^2 + x^2 =$ _____ $=$ _____ $=$ _____

1.3 Integrals involving $\sqrt{x^2 - a^2}$:

Let $x =$ _____,

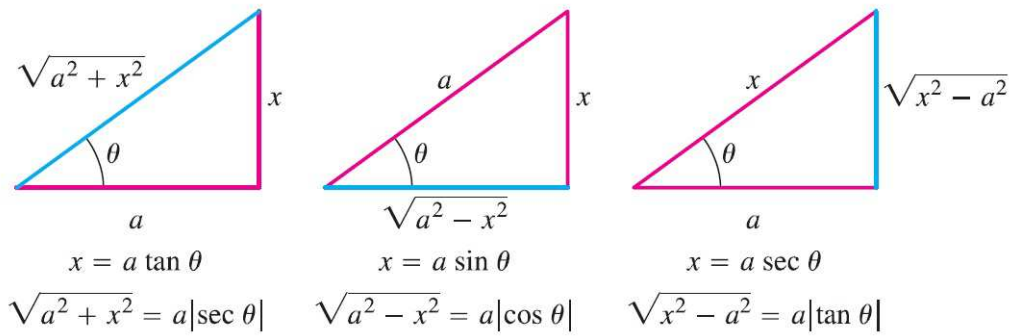
$x^2 - a^2 =$ _____ $=$ _____ $=$ _____

1.4 (a) $\sqrt{a^2 + x^2} =$ _____

(b) $\sqrt{a^2 - x^2} =$ _____

(c) $\sqrt{x^2 - a^2} =$ _____

圖示如下:



$x = a\tan\theta$ $\quad$ $x = a\sin\theta$ $\quad$ $x = a\sec\theta$

$\sqrt{a^2+x^2} = a|\sec\theta|$ $\quad$ $\sqrt{a^2-x^2} = a|\cos\theta|$ $\quad$ $\sqrt{x^2-a^2} = a|\tan\theta|$

FIGURE 8.2 Reference triangles for the three basic substitutions identifying the sides labeled x and a for each substitution.

✐ Ex. **1** ..... (`example1, p450`)

Evaluate $\displaystyle\int \frac{dx}{\sqrt{4+x^2}}$.

sol:

✐ Ex. **2** ..... (`example2, p451`)

Evaluate $\displaystyle\int \frac{x^2\,dx}{\sqrt{9-x^2}}$

sol:

✐ Ex. **3** ..... (`example3, p451`)

Evaluate $\displaystyle\int \frac{dx}{\sqrt{25x^2-4}}, \quad x > 2/5.$

sol:

實習課練習 (EXERCISE 8.3)

4. $\displaystyle\int_0^2 \frac{dx}{8+2x^2}$

7. $\displaystyle\int \sqrt{25-t^2}\ dt$

10. $\displaystyle\int \frac{5\ dx}{\sqrt{25x^2-9}}, \quad x > 3/5$

17. $\displaystyle\int \frac{x^3\ dx}{\sqrt{x^2+4}}$

20. $\displaystyle\int \frac{\sqrt{9-w^2}}{w^2}\ dw$

33. $\displaystyle\int \frac{v^2}{(1-v^2)^{5/2}}\ dv$

43. $\displaystyle\int \frac{x\ dx}{\sqrt{1+x^4}}$

46. $\displaystyle\int \sqrt{\frac{x}{1-x^3}}\ dx$

THOMAS' CALCULUS (12/E)

8.4 Integration of Rational Functions by Partial Fractions

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1 Partial Fractions

1.1 This section shows how to express a rational function (a quotient of polynomials) as a sum of simpler fractions, called _____,

Example: $\dfrac{5x-3}{x^2-2x-3} =$ _____

$\displaystyle\int \frac{5x-3}{x^2-2x-3}\,dx =$ _____ $=$ _____

1.2 The method of partial fractions: Example: $\dfrac{5x-3}{x^2-2x-3} =$ _____

✐ Ex. 1 ..... (`example1, p455`)

Use partial fractions to evaluate $\displaystyle\int \frac{x^2+4x+1}{(x-1)(x+1)(x+3)}\,dx$ (Distinct linear factors)

sol:

✐ Ex. **2** ..... (`example2, p455`)

Use partial fractions to evaluate $\int \frac{6x+7}{(x+2)^2}\,dx$ (A repeated linear factor)

sol:

✐ Ex. **3** ..... (`example3, p456`)

Use partial fractions to evaluate $\int \frac{2x^3-4x^2-x-3}{x^2-2x-3}\,dx$ (An improper fraction)

sol:

✐ Ex. **4** ..... (`example4, p456`)

Use partial fractions to evaluate $\int \frac{-2x+4}{(x^2+1)(x-1)^2}\,dx$ (An irreducible quadratic factor)

sol:

✐ Ex. **5** ..... (`example5, p457`)

Use partial fractions to evaluate $\displaystyle\int \frac{dx}{x(x^2+1)^2}\,dx$ (An repeated irreducible quadratic factor)

sol:

2 The Heaviside Method for Linear Factors

2.1 *Heaviside Method*

Write the partial-fraction expansion of $f(x)/g(x)$ as

$$\frac{f(x)}{g(x)} = \underline{\qquad\qquad\qquad\qquad\qquad\qquad},$$

where

$$A_1 = \underline{\qquad\qquad\qquad\qquad}$$

$$A_2 = \underline{\qquad\qquad\qquad\qquad\qquad\qquad}$$

$$\vdots$$

$$A_n = \underline{\qquad\qquad\qquad\qquad\qquad\qquad}$$

✐ Ex. **6** ..... (`example6, p458`)

Find A, B and C in the equation: $\dfrac{x^2+1}{(x-1)(x-2)(x-3)} = \dfrac{A}{x-1} + \dfrac{B}{x-2} + \dfrac{C}{x-3}$

sol:

✐ Ex. 7 ..... (example7, p459)

Use the Heaviside Method to evaluate $\displaystyle\int \frac{x+4}{x^3+3x^2-10x}\,dx$.

sol:

✐ Ex. 8 ..... (example8, p460)

Find A, B and C in the equation: $\dfrac{x-1}{(x+1)^3} = \dfrac{A}{x+1} + \dfrac{B}{(x+1)^2} + \dfrac{C}{(x+1)^3}$

sol:

✐ Ex. **9** ..... (`example9, p460`)

Find A, B and C in the expression: $\dfrac{x^2+1}{(x-1)(x-2)(x-3)} = \dfrac{A}{x-1} + \dfrac{B}{x-2} + \dfrac{C}{x-3}$

sol:

實習課練習 (EXERCISE 8.4)

☐ Expand the quotients by partial fractions.

2. $\dfrac{5x-7}{x^2-3x+2}$

5. $\dfrac{z+1}{z^2(z-1)}$

8. $\dfrac{t^4+9}{t^4+9t^2}$

. . . . . .

☐ Express the integrands as a sum of partial fractions and evaluate the integrals.

16. $\displaystyle\int \frac{x+3}{2x^3-8x}\,dx$

17. $\displaystyle\int_0^1 \frac{x^3\,dx}{x^2+2x+1}$

20. $\displaystyle\int \frac{x^2\,dx}{(x-1)(x^2+2x+1)}$

24. $\displaystyle\int \frac{8x^2+8x+2}{(4x^2+1)^2}\,dx$

33. $\displaystyle\int \frac{2x^3-2x^2+1}{x^2-x}\,dx$

39. $\displaystyle\int \frac{e^t\,dt}{e^{2t}+3e^t+2}$

42. $\displaystyle\int \frac{\sin\theta\,d\theta}{\cos^2\theta+\cos\theta-2}$

THOMAS' CALCULUS (12/E)

8.7 Improper Integrals

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1 Infinite Limits of Integration

1.1 *Definitions: Type I Improper Integrals*

Integrals with _____ of integration are **improper integrals of Type I**.

(a) If $f(x)$ is continuous on $[a, \infty)$, then

$$\int_a^{\infty} f(x)\, dx = \underline{\qquad\qquad\qquad}$$

(b) If $f(x)$ is continuous on $(-\infty, b]$, then

$$\int_{-\infty}^{b} f(x)\, dx = \underline{\qquad\qquad\qquad}$$

(c) If $f(x)$ is continuous on $(-\infty, \infty)$ then

$$\int_{-\infty}^{\infty} f(x)\, dx = \underline{\qquad\qquad\qquad\qquad\qquad}$$

where c is any real number. In each case, if the limit is _____ we say that the improper integral _____ and that the _____ is the **value** of the improper integral. If the limit fails to exist, the improper integral _____ (the area under the curve is infinite).

1.2 Examples:

(a) Upper limit: $\displaystyle\int_1^{\infty} \frac{\ln x}{x^2}\, dx = \underline{\qquad\qquad\qquad}$

(b) Lower limit: $\displaystyle\int_{-\infty}^{0} \frac{dx}{1+x^2} = \underline{\qquad\qquad\qquad}$

(c) Both limits: $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{1+x^2} =$ _____ .

Ex. 1 ..... (`example1, p479`)

Is the area under the curve $y = (\ln x)/x^2$ from $x = 1$ to $x = \infty$ finite? If so, what is it?

sol:

Ex. 2 ..... (`example2, p479`)

Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$.

sol:

Ex. 3 ..... (`example2, p480`)

For what values of p does the integral $\displaystyle\int_{1}^{\infty} \frac{1}{x^p}\, dx$ converge? When the integral does converge, what is its value?

sol:

2 Integrands with Vertical Asymptotes

2.1 *Definitions: Type II Improper Integrals*

Integrals of functions that become _____ within the interval of integration are **improper integrals of Type II**.

(a) If $f(x)$ is continuous on $(a, b]$ and is discontinuous at a then
$$\int_a^b f(x)\ dx = \underline{\qquad\qquad\qquad}$$

(b) If $f(x)$ is continuous on $[a, b)$ and is discontinuous at b, then
$$\int_a^b f(x)\ dx = \underline{\qquad\qquad\qquad}$$

(c) If $f(x)$ is discontinuous at c, where $a < c < b$, and continuous on $[a, c) \cup (c, b]$ then
$$\int_a^b f(x)\ dx = \underline{\qquad\qquad\qquad\qquad\qquad}$$

In each case, if the limit is finite we say the improper integral _____ and that the limit is the value of the improper integral. If the limit does not exist, the integral _____.

2.2 Examples:

(a) Upper endpoint: $\displaystyle\int_0^1 \frac{dx}{(x-1)^{2/3}} = \underline{\qquad\qquad\qquad}$

(b) Lower endpoint: $\displaystyle\int_1^3 \frac{dx}{(x-1)^{2/3}} = \underline{\qquad\qquad\qquad}$

(c) Interior point: $\displaystyle\int_0^3 \frac{dx}{(x-1)^{2/3}} = \underline{\qquad\qquad\qquad\qquad\qquad}$

✐ Ex. 4 ..... (`example4, p482`)

Investigate the convergence of $\displaystyle\int_0^1 \frac{1}{1-x}\ dx$.

sol:

✐ Ex. **5** ..... (`example5, p482`)

Evaluate $\displaystyle\int_0^3 \frac{dx}{(x-1)^{2/3}}$.

sol:

3 Tests for Convergence and Divergence

3.1 *Theorem 1: Direct Comparison Test*

Let f and g be continuous on $[a, \infty)$ with _____ for all $x \geq a$. Then

(a) $\displaystyle\int_a^\infty f(x)\,dx$ _____ if $\displaystyle\int_a^\infty g(x)\,dx$ _____.

(b) $\displaystyle\int_a^\infty g(x)\,dx$ _____ if $\displaystyle\int_a^\infty f(x)\,dx$ _____.

3.2 *Theorem 2: Limit Comparison Test*

If the positive functions f and g are continuous on $[a, \infty)$ and if _____, $0 < L < \infty$, then _____ and _____ both _____ or both _____.

✐ Ex. **6** ..... (`example6, p483`)

Does the integral $\displaystyle\int_1^\infty e^{-x^2}\,dx$ converges?

sol:

✐ Ex. **7** ..... (`example7, p484`)

(a) $\displaystyle\int_1^\infty \frac{\sin^2 x}{x^2}\,dx$ converges because

(b) $\displaystyle\int_1^\infty \frac{1}{\sqrt{x^2-0.1}}\,dx$ diverges because

✐ Ex. **8** ..... (`example8, p485`)

Show that $\displaystyle\int_1^\infty \frac{dx}{1+x^2}$ converges and find the integral value.

sol:

✐ Ex. **9** ..... (`example9, p485`)

Investigate the convergence of $\displaystyle\int_1^\infty \frac{1-e^{-x}}{x}\,dx$.

sol:

實習課練習 (EXERCISE 8.7)

☐ Evaluate the integrals.

2. $\displaystyle\int_1^\infty \frac{dx}{x^{1.001}}$

7. $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

10. $\displaystyle\int_{-\infty}^2 \frac{2\ dx}{x^2+4}$

13. $\displaystyle\int_{-\infty}^\infty \frac{2x\ dx}{(x^2+1)^2}$

17. $\displaystyle\int_0^\infty \frac{dx}{(1+x)\sqrt{x}}$

22. $\displaystyle\int_0^\infty 2e^{-\theta}\sin\theta\ d\theta$

25. $\displaystyle\int_0^1 x\ln x\ dx$

30. $\displaystyle\int_2^4 \frac{dt}{t\sqrt{t^2-4}}$

32. $\displaystyle\int_0^2 \frac{dx}{\sqrt{|x-1|}}$

☐ Test the integrals for convergence.

39. $\displaystyle\int_0^{\ln x} x^{-2}e^{-1/x}dx$

45. $\displaystyle\int_{-1}^1 \ln|x|\ dx$

54. $\displaystyle\int_2^\infty \frac{x\ dx}{\sqrt{x^4-1}}$

58. $\displaystyle\int_2^\infty \frac{1}{\ln x}\ dx$

61. $\displaystyle\int_1^\infty \frac{1}{\sqrt{e^x-x}}$

THOMAS' CALCULUS (12/E)

10.1 Sequences

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1 Sequences, Convergence and Divergence

1.1 A sequence is _____(_____) in a given order.

1.2 Each of $a_1, a_2, \cdots$ are the _____ of the sequence.

1.3 Example: $2, 4, 6, 8, 10, 12, \cdots, 2n, \cdots$ has first term $a_1 = 2$, second term $a_2 = 4$ and _____ (_____). The integer n is called the _____ of a_n.

1.4 *Definitions: Infinite Sequence*

> An infinite sequence of numbers is a _____ whose _____ is the set of _____.

1.5 The sequence $1, 2, 3, 4, \cdots$ is not the same as the sequence $2, 1, 3, 4, \cdots$; _____ is important.

1.6 Examples:

nth term	listing terms	write
$a_n = \sqrt{n}$	$\{a_n\} =$ _____	$\{a_n\} =$ _____
$b_n = (-1)^{n+1}\dfrac{1}{n}$	_____	_____
$c_n = \dfrac{n-1}{n}$	_____	_____
$d_n = (-1)^{n+1}$	_____	_____

1.7 *Definitions: Converges, Diverges, Limit*

The sequence $\{a_n\}$ converges to the number _____ if to every positive number _____ there corresponds an integer _____ such that for all _____,

_____

If no such number L exists, we say that $\{a_n\}$ _____. If $\{a_n\}$ converges to L, we write _____ or simply _____ and call L the _____ of the sequence.

1.8 Examples:

(a) $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots, \frac{1}{n}, \cdots\}$; $\lim_{n\to\infty} a_n =$ _____

(b) $\{0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \cdots, 1 - \frac{1}{n}, \cdots\}$; $\lim_{n\to\infty} a_n =$ _____

(c) $\{\sqrt{1}, \sqrt{2}, \sqrt{3}, \cdots, \sqrt{n}, \cdots\}$; _____

(d) $\{1, -1, 1, -1, \cdots, (-1)^{n+1}, \cdots\}$; _____

1.9 *Definitions: Diverges to Infinity*

The sequence $\{a_n\}$ _____ to infinity if for every number _____ there is an integer _____ such that for all _____ larger than N, _____. If this condition holds we write

_____ or _____

Similarly if for every number m there is an integer N such that for all $n > N$ we have $a_n < m$ then we say $\{a_n\}$ diverges to negative infinity and write

_____ or _____

1.10 A sequence may diverge without diverging to infinity or negative infinity. Examples: _____ and _____.

2 Calculating Limits of Sequences

2.1 *Theorem 1*

Let $\{a_n\}$ and $\{b_n\}$ be sequences of real numbers and let A and B be real numbers. The following rules hold if $\lim_{n\to\infty} a_n = A$ and $\lim_{n\to\infty} b_n = B$

1. Sum Rule: _____
2. Difference Rule: _____

3. Product Rule: _____
4. Constant Multiple Rule: _____
5. Quotient Rule: _____

2.2 *Theorem 2: The Sandwich Theorem for Sequences*

Let $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ be sequences of real numbers. If _____ holds for all n beyond some index N, and if $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n =$ _____ then _____ also.

2.3 *Theorem 3: The Continuous Function Theorem for Sequences*

Let $\{a_n\}$ be a sequence of real numbers. If _____ and if f is a function that is _____ and defined at all a_n, then _____.

Ex. 1 ..... (`example3, p536`)

(a) $\lim_{n\to\infty} (-\frac{1}{n}) =$

(b) $\lim_{n\to\infty} (\frac{n-1}{n}) =$

(c) $\lim_{n\to\infty} \frac{5}{n^2} =$

(d) $\lim_{n\to\infty} \frac{4-7n^6}{n^6+3} =$

Ex. 2 ..... (`example4, p536`)

(a) $\lim_{n\to\infty} (\frac{\cos n}{n}) =$

(b) $\lim_{n\to\infty} (\frac{1}{2^n}) =$

(c) $\lim_{n\to\infty} (-1)^n \frac{1}{n} =$

✐ Ex. **3** ..... (`example5, p537`)

Show that $\sqrt{(n+1)/n} \to 1$.

sol:

✐ Ex. **4** ..... (`example6, p537`)

Find $\lim_{n\to\infty} 2^{1/n}$.

sol:

3 Using L'Hopital's Rule

3.1 *Theorem 4*

> Suppose that $f(x)$ is a function defined for all $x \geq n_0$ and that is a sequence of real numbers such that _____ for $n \geq n_0$. Then
> $\Rightarrow$ _____ _____

✐ Ex. **5** ..... (`example7, p537`)

Show that $\lim_{n\to\infty} \frac{\ln n}{n} = 0$.

sol:

✐ Ex. **6** ..... (`example8, p537`)

Does the sequence whose nth term is $a_n = (\frac{n+1}{n-1})^n$ converges? If so, find $\lim_{n\to\infty} a_n$.

sol:

4 Commonly Occurring Limits

4.1 *Theorem 5*

1. $\lim\limits_{n\to\infty} \frac{\ln n}{n} =$ _____
2. $\lim\limits_{n\to\infty} \sqrt[n]{n} =$ _____
3. $\lim\limits_{n\to\infty} x^{1/n} =$ _____ $(x > 0)$
4. $\lim\limits_{n\to\infty} x^n =$ _____ $(|x| < 1)$
5. $\lim\limits_{n\to\infty} (1 + \frac{x}{n})^n =$ _____ (any x)
6. $\lim\limits_{n\to\infty} \frac{x^n}{n!} =$ _____ (any x)

4.2 *Definition*

A sequence $\{a_n\}$ is _____ if _____ for all n. That is, _____. The sequence is _____ if _____ for all n. The sequence $\{a_n\}$ is _____ if it is either nondecreasing or nonincreasing.

4.3 *Theorem 6: The Monotonic Sequence Theorem*

If a sequence $\{a_n\}$ is both _____ and _____, then the sequence _____.

✐ Ex. **7** ..... (example9, p538)

(a) $\lim_{n\to\infty} \frac{\ln(n^2)}{n} =$

(b) $\lim_{n\to\infty} \sqrt[n]{n^2} =$

(c) $\lim_{n\to\infty} \sqrt[n]{3n} =$

(d) $\lim_{n\to\infty} (-\frac{1}{2})^n =$

(e) $\lim_{n\to\infty} (\frac{n-2}{n})^n =$

(f) $\lim_{n\to\infty} \frac{100^n}{n!} =$

實習課練習 (EXERCISE 10.1)

☐ Find the values of a_1, a_2, a_3 and a_4.

3. $a_n = \dfrac{(-1)^{n+1}}{2n-1}$

☐ Write out the first ten terms of the sequence.

9. $a_1 = 2, a_{n+1} = (-1)^{n+1} a_n/2$

☐ Find a formula for the nth term of the sequence.

16. The sequence $1, -\dfrac{1}{4}, \dfrac{1}{9}, -\dfrac{1}{16}, \dfrac{1}{25}, \cdots$

19. The sequence $0, 3, 8, 15, 24, \cdots$

☐ Which of the sequence $\{a_n\}$ converge, and which diverge? Find the limit of each convergent sequence.

27. $a_n = 2 + (0.1)^n$

31. $a_n = \dfrac{1-5n^4}{n^4+8n^3}$

36. $a_n = (-1)^n(1-\dfrac{1}{n})$

43. $a_n = \sin(\dfrac{\pi}{2} + \dfrac{1}{n})$

54. $a_n = (1-\dfrac{1}{n})^n$

57. $a_n = (\dfrac{3}{n})^{1/n}$

63. $a_n = \dfrac{n!}{n^n}$

71. $a_n = (\dfrac{x^n}{2n+1})^{1/n}, \quad x > 0$

84. $a_n = \sqrt[n]{n^2+n}$

86. $a_n = \dfrac{(\ln n)^5}{\sqrt{n}}$

89. $a_n = \dfrac{1}{n}\displaystyle\int_1^n \dfrac{1}{x}\, dx$

THOMAS' CALCULUS (12/E)

10.2 Infinite Series

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1 Series

1.1 *Definitions: Infinite Series, nthTerm, Partial Sum, Converges, Sum*

(a) Given a sequence of numbers $\{a_n\}$, an expression of the form

$$\underline{\qquad\qquad\qquad\qquad\qquad\qquad}$$

is an _____. The number a_n is the _____ of the series.

(b) The sequence $\{s_n\}$ defined by

$$s_n = \underline{\qquad\qquad\qquad} = \underline{\qquad\qquad}$$

is the sequence of _____ of the series, the number being the _____ partial sum.

(c) If _____ converges to a limit L, we say that _____ and that its sum is _____. In this case, we also write

$$a_1 + a_2 + \cdots + a_n + \cdots = \underline{\qquad\qquad\qquad\qquad}$$

(d) If the sequence of partial sums of the series does not converge, we say that the series _____.

1.2 The series of the form

$$\underline{\qquad\qquad\qquad\qquad\qquad\qquad} = \qquad\qquad = \qquad\qquad$$

is called _____, in which a and r are fixed real number and $a \neq 0$.

(a) Examples: $a = 1$

i. $r = \frac{1}{2}$,

ii. $r = -\frac{1}{3}$,

iii. $r = 1$

(b) $s_n =$

(c) If $|r| < 1$, the geometric series converges: , $|r| < 1$.

(d) If $|r| \geq 1$, the series _____. _____

Ex. 1 ..... (example1, p546)

$$\sum_{n=1}^{\infty} \frac{1}{9}(\frac{1}{3})^{n-1} =$$

Ex. 2 ..... (example2, p546)

$$\sum_{n=0}^{\infty} \frac{(-1)^n 5}{4^n} =$$

Ex. 3 ..... (example4, p547)

Express the repeating decimal 5.232323... as the ratio of two integers.

sol:

✐ Ex. 4 ..... (example5, p547)

Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

sol:

2 The nth-Term Test for Divergence

2.1 *Theorem 7*

If _____ converges, then _____.

2.2 *The nth-Term Test for Divergence*

$\sum_{n=1}^{\infty} a_n$ diverges if _____ fails to _____ or is different from _____.

2.3 *Theorem 8*

If $\sum a_n = A$ and $\sum b_n = B$ are convergent series, then

(a) Sum Rule: _____

(b) Difference Rule: _____

(c) Constant Multiple Rule: _____, (any number k)

2.4 $\sum(a_n + b_n)$ can converge when $\sum a_n$ and $\sum b_n$ diverge.
Example: $\sum a_n =$ _____, $\sum b_n =$ _____,
$\sum(a_n + b_n) =$ _____

2.5 Adding or Deleting Terms: we can add or delete a _____ number of terms without altering the series'convergence or divergence, although in the case of convergence this will usually change the sum.

2.6 Reindexing:

(a) $\sum_{n=1}^{\infty} a_n =$ _____

(b) If $\sum_{n=1}^{\infty} a_n$ converges, then _____ converges for any $k > 1$.

(c) If $\sum_{n=k}^{\infty} a_n$ converges for any $k > 1$ then _____ converges.

(d) $\sum_{n=1}^{\infty} a_n =$ _____ $=$ _____ $=$ _____

(e) Example:

$$\sum_{n=1}^{\infty} \frac{1}{2^{n-1}} = \underline{\qquad\qquad} = \underline{\qquad\qquad} = \underline{\qquad\qquad}$$

✐ Ex. **5** ..... (`example7, p548`)

Test the divergences of the series.

(a) $\displaystyle\sum_{n=1}^{\infty} n^2$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{-n}{2n+5}$

✐ Ex. **6** ..... (`example9, p550`)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n-1}-1}{6^{n-1}} =$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{4}{2^n} =$

實習課練習 (EXERCISE 10.2)

☐ Find the sum of the series if the series converges.

7. $\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n}$.

13. $\sum_{n=0}^{\infty} (\frac{1}{2^n} + \frac{(-1)^n}{5^n})$.

44. $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$.

45. $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$

47. $\sum_{n=1}^{\infty} \left(\frac{1}{\ln(n+2)}) - \frac{1}{\ln(n+1)}\right)$

☐ Express each of the number as the ratio of two integers.

20. $0.\overline{234} = 0.234234234...$

25. $1.24\overline{123} = 1.24123123...$

☐ Use the nth-Term Test for divergence to show that the series is divergent, or state that the test is inconclusive.

30. $\sum_{n=1}^{\infty} \frac{n}{n^2+3}$

33. $\sum_{n=1}^{\infty} \ln \frac{1}{n}$

☐ Which series converge and which diverge? If a series converges, find its sum.

49. $\sum_{n=0}^{\infty} (\frac{1}{\sqrt{2}})^n$

54. $\sum_{n=0}^{\infty} \frac{\cos n\pi}{5^n}$

58. $\sum_{n=0}^{\infty} \frac{1}{x^n}, \quad |x| > 1.$

60. $\sum_{n=1}^{\infty}(1-\frac{1}{n})^n$

67. $\sum_{n=0}^{\infty}(\frac{e}{\pi})^n$

☐ Find the values of x for which the given geometric series converges.

75. $\sum_{n=0}^{\infty}(-1)^n(x+1)^n$

78. $\sum_{n=0}^{\infty}(\ln x)^n$

THOMAS' CALCULUS (12/E)

10.3 The Integral Test

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1 Nondecreasing Partial Sums

1.1 Given a series we have two questions: (a) Does the series _____? (b) If it converges, what is its _____?

1.2 Suppose that $\sum_{n=1}^{\infty} a_n$ is an infinite series with $a_n \geq 0$ for all n. Then each partial sum is greater than or equal to its predecessor because _____. The partial sums form a _____: _____

1.3 *Corollary of Theorem 6*

> A series $\sum_{n=1}^{\infty} a_n$ of nonnegative terms converges if and only if its _____ are _____.

1.4 The _____

$$\sum_{n=1}^{\infty} \frac{1}{n} = \underline{\hspace{6cm}}$$

is divergent, but this doesn't follow from the nth-Term Test.
The nth term $1/n$ does go to zero, but the series still diverges. The reason it diverges is because there is no upper bound for its partial sums.

1.5 *Theorem 9: The Integral Test*

> Let $\{a_n\}$ be a sequence of _____. Suppose that _____, where f is a _____, _____, _____ function of x for all $x \geq N$ (N a positive integer). Then the series _____ and the integral _____ both converge or both diverge.

(說明如下:)

✐ Ex. **1** ..... (`example2, p553`)

Does $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges?

sol:

✐ Ex. **2** ..... (`example3, p555`)

Show that the **p-series**

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} + \cdots$$

converges if $p > 1$, and diverges if $p \leq 1$.

sol:

✐ Ex. **3** ..... (`example4, p555`)

Does $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converge?

sol:

實習課練習 (EXERCISE 10.3)

☐ Use the Integral Test to determine if the series converge or diverge.

3. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+4}$.

6. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$.

9. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{e^{n/3}}$.

☐ Determining convergence or divergence.

15. $\displaystyle\sum_{n=1}^{\infty} \frac{3}{\sqrt{n}}$.

19. $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n}$.

22. $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{4^n+3}$.

27. $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{n}}{\ln n}$.

28. $\displaystyle\sum_{n=1}^{\infty} (1+\frac{1}{n})^n$.

30. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(\ln 3)^n}$.

36. $\displaystyle\sum_{n=1}^{\infty} \frac{2}{1+e^n}$.

38. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+1}$.

THOMAS' CALCULUS (12/E)

10.4 Comparison Tests

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1 Comparison Tests

1.1 *Theorem 10: The Comparison Test*

Let $\sum a_n$ be a series with _____ terms.

(a) $\sum a_n$ _____ if there is a _____ series $\sum c_n$ with _____ for all $n > N$, for some integer N.

(b) $\sum a_n$ _____ if there is a _____ series of nonnegative terms $\sum d_n$ with _____ for all $n > N$, for some integer N.

Ex. 1 ..... (`example1, p559`)

(a) Does $\displaystyle\sum_{n=1}^{\infty} \frac{5}{5n-1}$ converge?

(b) Does $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n!}$ converge?

(c) Does $5 + \dfrac{2}{3} + \dfrac{1}{7} + 1 + \dfrac{1}{2+\sqrt{1}} + \cdots + \dfrac{1}{2^n + \sqrt{n}} + \cdots$ converge?

2 The Limit Comparison Test

2.1 *Theorem 11: Limit Comparison Test*

Suppose that $a_n > 0$ and $b_n > 0$ for all $n \geq N$ (N an integer).

(a) If _____ then _____ both converge or both diverge.

(b) If _____ and _____ converges, then _____ converges.

(c) If _____ and _____ diverges, then _____ diverges.

Ex. **2** ..... (`example2, p560`)

(a) Does $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{(n+1)^2}$ converge?

(b) Does $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$ converge?

(c) Does $\displaystyle\sum_{n=2}^{\infty} \frac{1 + n\ln n}{n^2 + 5}$ converge?

✐ Ex. **3** ..... (`example3, p561`)

Does $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$ converge?

sol:

實習課練習 (EXERCISE 10.4)

☐ Use the Comparison Test to determine if each series converges or diverges.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+30}$.

6. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n3^n}$.

☐ Use the Limit Comparison Test to determine if each series converges or diverges.

10. $\displaystyle\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n^2+2}}$.

14. $\displaystyle\sum_{n=1}^{\infty} (\frac{2n+3}{5n+4})^n$.

☐ Determining convergence or divergence?

17. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n}+\sqrt[3]{n}}$

19. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2 n}{2^n}$

28. $\displaystyle\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^3}$

34. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$

38. $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n-1}+1}{3^n}$

51. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt[n]{n}}$

53. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1+2+3+\cdots+n}$

THOMAS' CALCULUS (12/E)

10.5 The Ratio and Root Tests

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1 The Ratio Test

1.1 *Theorem 12: The Ratio Test*

Let $\sum a_n$ be a series with positive terms and suppose that _____
Then

(a) the series converges if _____,

(b) the series diverges if _____ or _____,

(c) the test is inconclusive if _____.

Ex. 1 ..... (example1, p564)

Investigate the convergence of the series.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n+5}{3^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{n!n!}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{4^n n! n!}{(2n)!}$

2 The Root Test

2.1 *Theorem 13: The Root Test*

Let $\sum a_n$ be a series with $a_n \geq 0$ and suppose that _____ Then

(a) the series converges if _____,

(b) the series diverges if _____ or _____,

(c) the test is inconclusive if _____.

Ex. **2** ..... (`example3, p566`)

Investigate the convergence of the series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^2}$

(c) $\displaystyle\sum_{n=1}^{\infty} (\frac{1}{1+n})^n$

實習課練習 (EXERCISE 10.5)

☐ Use the Ratio Test to determine if each series converges or diverges.

3. $\displaystyle\sum_{n=1}^{\infty} \frac{(n-1)!}{(n+1)^2}$.

6. $\displaystyle\sum_{n=2}^{\infty} \frac{3^{n+2}}{\ln n}$.

☐ Use the Root Test to determine if each series converges or diverges.

12. $\displaystyle\sum_{n=1}^{\infty} \left(\ln\left(e^2 + \frac{1}{n}\right)\right)^{n+1}$.

13. $\displaystyle\sum_{n=1}^{\infty} \frac{8}{(3+(1/n))^{2n}}$.

☐ Determining convergence or divergence.

18. $\displaystyle\sum_{n=1}^{\infty} n^2 e^{-n}$

20. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{10^n}$

23. $\displaystyle\sum_{n=1}^{\infty} \frac{2+(-1)^n}{1.25^n}$

27. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$

32. $\displaystyle\sum_{n=1}^{\infty} \frac{n \ln n}{2^n}$

36. $\displaystyle\sum_{n=1}^{\infty} \frac{n 2^n (n+1)!}{3^n n!}$

38. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

THOMAS' CALCULUS (12/E)

10.6 Alternating Series, Absolute and Conditional Convergence

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1 Alternating Series

1.1 A series in which the terms are alternately _____ and _____ is an _____.

(a) alternating harmonic series: _____

(b) geometric series with $r = -1/2$: _____

(c) $1 - 2 + 3 - 4 + 5 - 6 + \cdots + (-1)^{n+1}n + \cdots$

1.2 *Theorem 14: The Alternating Series Test (Leibniz's Theorem)*

The series

$$\sum_{n=1}^{\infty}(-1)^{n+1}u_n = u_1 - u_2 + u_3 - u_4 + \cdots$$

converges if all three of the following conditions are satisfied:

(a) _____ are all _____.

(b) _____ for all $n \geq N$, for some integer N.

(c) _____.

✐ Ex. **1** ..... (`example1, p569`)

Does the alternating harmonic series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$ converge?

sol:

2 Absolute and Conditional Convergence

2.1 *Definitions: Absolutely Convergent*

> A series _____ (is _____) if the corresponding series of absolute values, _____

2.2 *Definitions: Conditionally Convergent*

> A series that converges but does not converge absolutely converges _____.

2.3 The geometric series $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\cdots$ converges _____ because _____ converges.

2.4 The alternating hramonic series does not converge absolutely. It converges _____.

2.5 *Theorem 16: The Absolute Convergence Test*

> If _____ converges, then _____ converges.

✐ Ex. **2** ..... (`example4, p571`)

(a) Does $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^2}$ converge absolutely?

(b) Does $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ converge absolutely?

✎ Ex. **3** ..... (`example5, p571`)

Determine the convergence (absolutely convergence, conditionally convergence) of the alternating p-series: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p}$.

sol:

3 Summary of Tests for Convergence and Divergence of Series

3.1 The nth-Term Test: _____.

3.2 Geometric series: _____.

3.3 p-series: _____.

3.4 Series with nonnegative terms:
Try _____.
Try _____.

3.5 Series with some negative terms: _____.

3.6 Alternating series: _____.

實習課練習 (EXERCISE 10.6)

☐ Determining convergence or divergence.

2. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^{3/2}}$

11. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\ln n}{n}$

13. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sqrt{n}+1}{n+1}$

☐ Which of the series converge absolutely, which converge and which diverge?

16. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(0.1)^n}{n}$

20. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n!}{2^n}$

25. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1+n}{n^2}$

30. $\displaystyle\sum_{n=2}^{\infty}(-1)^{n}\frac{\ln n}{n-\ln n}$

39. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\frac{(2n)!}{2^n n! n}$

42. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}(\sqrt{n^2+n}-n)$

44. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\frac{1}{\sqrt{n}+\sqrt{n+1}}$

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10.7 Power Series

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1 Power Series, The Convergence Theorem

1.1 *Definitions: Power Series, Center, Coefficients*

A power series about _____ is a series of the form

_____

A power series about _____ is a series of the form

_____

in which the center a and the coefficients $c_0, c_1, c_2, \cdots, c_n$ are constants.

2 The Radius of Convergence of a Power Series

2.1 *Theorem 18: The Convergence Theorem for Power Series*

If the power series $\sum_{n=0}^{\infty} a_n x^n$ converges for $x = c \neq 0$,then it converges absolutely for all x with _____. If the series diverges for $x = d$, then it diverges for all x with _____.

2.2 *COROLLARY TO THEOREM 18*

The convergence of the series $\sum c_n(x-a)^n$ is the one of the following three:

(a) There is a positive number R such that the series diverges for x with _____ but converges absolutely for x with _____. The series may or may not converge at either of the endpoints _____ and _____.

(b) The series converges absolutely for _____.

(c) The series converges at _____ and diverges _____.

2.3 R is called the _____ of the power series and the interval of radius R centered at $x = a$ is called the interval of convergence.

2.4 *How to Test a Power Series for Convergence*

(a) Use the _____ Test (or _____ Test) to find the interval where the series converges _____. Ordinarily, this is an open interval _____ *or* _____

(b) If the interval of absolute convergence is _____, test for convergence or divergence at _____. Use a _____ Test, the _____ Test, or the _____ Test.

(c) If the interval of absolute convergence is _____, the series diverges for _____ (it does not even converge conditionally), because the nth term does not approach zero for those values of x.

Ex. 1 ..... (`example1, p575`)

Find the sum of a geometric power series with $a = 1$, $r = x$:

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \cdots + x^n + \cdots$$

sol:

Ex. 2 ..... (example2, p576)

Find the sum of a geometric power series with $a = 1$, $r = -(x-2)/2$:
$1 - \frac{1}{2}(x-2) + \frac{1}{4}(x-2)^2 + \cdots + (-\frac{1}{2})^n(x-2)^n + \cdots$

sol:

Ex. 3 ..... (example3(a), p576)

For what values of x do the power series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}$ converge?

sol:

Ex. 4 ..... (example3(b), p577)

For what values of x do the power series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^{2n-1}}{2n-1}$ converge?

sol:

Ex. 5 ..... (example3(c), p577)

For what values of x do the power series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ converge?

sol:

Ex. 6 ..... (example3(d), p577)

For what values of x do the power series $\sum_{n=0}^{\infty} n! x^n$ converge?

sol:

實習課練習 (EXERCISE 10.7)

☐ Find the series's radius and interval of convergence. Foe what values of x does the series converge absolutely, or conditionally?

3. $\sum_{n=0}^{\infty}(-1)^n(4x+1)^n$

8. $\sum_{n=0}^{\infty}\frac{(-1)^n(x+2)^n}{n}$

11. $\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{n!}$

15. $\sum_{n=0}^{\infty}\frac{x^n}{\sqrt{n^2+3}}$

23. $\sum_{n=1}^{\infty}(1+\frac{1}{n})^n x^n$

34. $\sum_{n=1}^{\infty}\frac{3\cdot 5\cdot 7\cdots(2n+1)}{n^2\cdot 2^n}x^{n+1}$

36. $\sum_{n=0}^{\infty}(\sqrt{n+1}-\sqrt{n})(x-3)^n$

THOMAS' CALCULUS (12/E)

10.8 Taylor and Maclaurin Series

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1 Series Representations

1.1 Assume that $f(x)$ is the sum of a power series

$$\begin{aligned} f(x) &= \underline{\qquad\qquad\qquad} \\ &= \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad} \end{aligned}$$

$$\begin{aligned} f'(x) &= \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad} \\ f"(x) &= \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad} \\ f'''(x) &= \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad} \\ &\vdots \\ f^{(n)}(x) &= \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad} \end{aligned}$$

1.2 $f^{(n)}(a) = \underline{\qquad\qquad} \Rightarrow \underline{\qquad\qquad\qquad}$

1.3 $f(x) =$ $\underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$

1.4 *Definitions: Taylor Series, Maclaurin Series*

> Let f be a function with derivatives of all orders throughout some interval containing a as an interior point.
>
> (a) Then the $\underline{\qquad\qquad\qquad}$ generated by f at $x = a$ is $\underline{\qquad\qquad\qquad}$
>
> $\underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$

(a) The _____ generated by f is _____ the Taylor series generated by f at $x = 0$.

✐ Ex. 1 ..... (`example1, p585`)

Find the Taylor series generated by $f(x) = 1/x$ at $a = 2$. Where, if anywhere, does the series converge to $1/x$?

sol:

2 Taylor Polynomials

2.1 The linearization of a differentiable function f at a point a is the polynomial of degree one given by

$$P_1(x) = \underline{\qquad\qquad\qquad}$$

We used this _____ to approximate $f(x)$ at values of x near a.

2.2 *Definitions: Taylor Polynomial of Order n*

Let f be a function with derivatives of order k for $k = 1, 2, \cdots, N$ in some interval containing a as an interior point. Then for any integer n from 0 through N, the _____ generated by f at $x = a$ is the polynomial

$P_n(x) =$ _____

Ex. 2 ..... (`example2, p586`)

Find the Taylor series and the Taylor polynomials generated by $f(x) = e^x$ at $x = 0$.

sol:

Ex. 3 ..... (`example3, p587`)

Find the Taylor series and the Taylor polynomials generated by $f(x) = \cos x$ at $x = 0$.

sol:

實習課練習 (EXERCISE 10.8)

☐ Find the Taylor polynomials of order 0,1,2, and 3 generated by f at a.

5. $f(x) = \dfrac{1}{x},\ a = 2$

10. $f(x) = \sqrt{1-x},\ a = 0$

☐ Find the Maclaurin series.

12. xe^x

15. $\sin 3x$

☐ Find the Taylor series generated by f at $x = a$.

23. $f(x) = x^3 - 2x + 4,\ a = 2$

27. $f(x) = 1/x^2,\ a = 1$

30. $f(x) = 2^x,\ a = 1$

THOMAS' CALCULUS (12/E)

11.3 Polar Coordinates

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1 Definition of Polar Coordinates

1.1 *Definitions: Polar Coordinates*

> Fix an _____ (called the _____) and an initial _____ from O. Then each point P can be located by assigning to it a _____ pair _____ in which r gives the _____ from O to P and θ gives the _____ from the initial ray to ray $\bar{OP}$.

1.2 圖示如下:

(a) Polar coordinates (b) (r, θ) are not unique. (c) (r, θ) can have negative r-values.

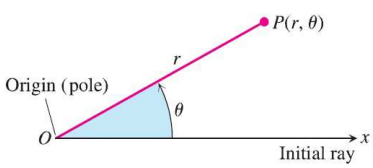

FIGURE 11.18 To define polar coordinates for the plane, we start with an origin, called the pole, and an initial ray.

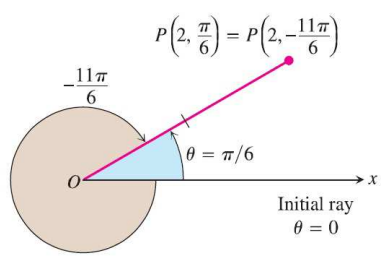

FIGURE 11.19 Polar coordinates are not unique.

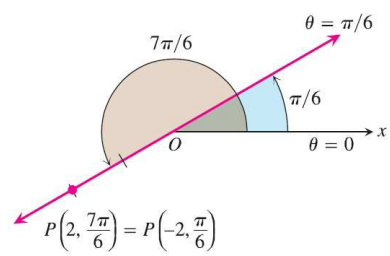

FIGURE 11.20 Polar coordinates can have negative r-values.

1.3 One pair of cartesian coordinates imples _____ pairs of polar coordinates.

1.4 Polar Equations and Graphs (I)

(a) If we hold r fixed at a constant value _____, the point _____ will lie _____ units from the origin O.

(b) As θ varies over any interval of length _____, P then traces a circle of _____ centered at O.

(c) If we hold θ fixed at a constant value _____ and let r vary between _____, the point $P(r, \theta)$ traces the line through O that makes an angle of measure θ_0 with the initial ray.

1.5 Polar Equations and Graphs (II)

(a) Equation: $r = a$, Graph: _____

(b) Equation: $\theta = \theta_0$, Graph: _____

(c) $r = 1$ and $r = -1$ are equations for the _____.

Ex. 1 ..... (`example1, p627`)

Find all the polar coordinates of the point $P(2, \pi/6)$

sol:

Ex. 2 ..... (`example3, p628`)

Graph the sets of points whose polar coordinates satisfy the following conditions.

(a) $1 \leq r \leq 2$ and $0 \leq \theta \leq \dfrac{\pi}{2}$

(b) $-3 \leq r \leq 2$ and $\theta = \dfrac{\pi}{4}$

(d) $\dfrac{2\pi}{3} \leq \theta \leq \dfrac{5\pi}{6}$

2 Relating Polar and Cartesian Coordinates

2.1 *Equations Relating Polar and Cartesian Coordinates*

$$x = r\cos\theta, \quad y = r\sin\theta, \quad x^2 + y^2 = r^2$$

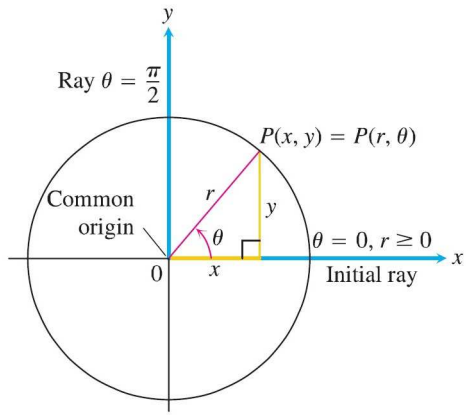

FIGURE 11.24 The usual way to relate polar and Cartesian coordinates.

Ex. 3 ..... (example5, p629)

Find a polar equation for the circle $x^2 + (y-3)^2 = 9$.

sol:

Ex. 4 ..... (example6, p629)

Replace the following polar equations by equivalent Cartesian equations, and identify their graphs.

(a) $r\cos\theta = -4$

(b) $r^2 = 4r\cos\theta$

(c) $r = \dfrac{4}{2\cos\theta - \sin\theta}$

sol:

實習課練習 (EXERCISE 11.3)

☐ Plot the following points given in a polar coordinates. Then find all the polar coordinates of each point.

3. $(2, \pi/2)$, $(2, 0)$, $(-2, \pi/2)$, $(-2, 0)$.

4. $(3, \pi/4)$, $(-3, \pi/4)$, $(3, -\pi/4)$, $(-3, -\pi/4)$.

☐ Replace the following polar equation by equivalent Cartesian equations.

30. $r\cos\theta = 0$

33. $r\cos\theta + r\sin\theta = 1$

38. $r^2 \sin 2\theta = 2$

44. $\cos^2\theta = \sin^2\theta$

49. $r = 2\cos\theta + 2\sin\theta$

☐ Replace the following Cartesian equation by equivalent polar equations.

54. $y = 1$

57. $x^2 + y^2 = 4$

60. $xy = 2$

62. $x^2 + xy + y^2 = 1$

65. $(x-3)^2 + (y+1)^2 = 4$

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11.4 Graphing in Polar Coordinates

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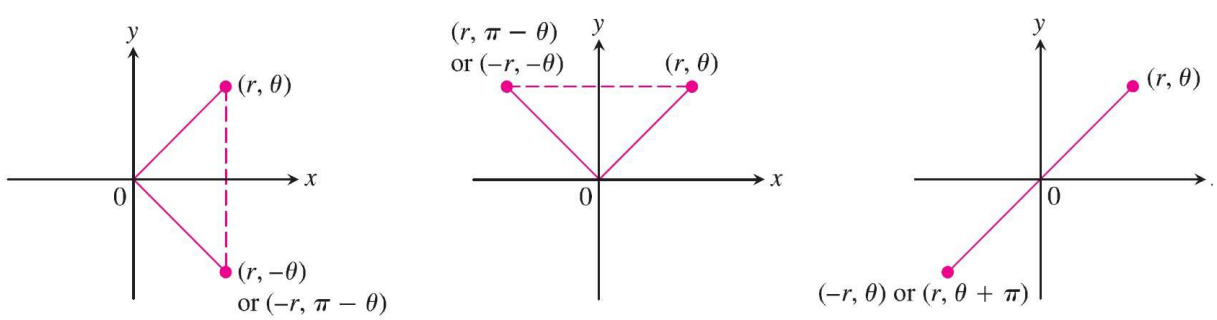
1 Symmetry and Slope

1.1 *Symmetry Tests for Polar Graphs*

(a) 1. Symmetry about the _____:
If the point (r, θ) lies on the graph, the point _____ or _____ lies on the graph.

(b) 2. Symmetry about the _____:
If the point (r, θ) lies on the graph, the point _____ or _____ lies on the graph.

(c) 3. Symmetry about the _____:
If the point (r, θ) lies on the graph, the point _____ or _____ lies on the graph.



(a) About the x-axis (b) About the y-axis (c) About the origin

1.2 *Slope*

The slope of a polar curve $r = f(\theta)$ is given by _____, not by _____.

1.3 *Slope of the Curve* $r = f(\theta)$

$$\left.\frac{dy}{dx}\right|_{(r,\theta)} = \underline{\qquad\qquad\qquad\qquad}, \text{ provided } dy/d\theta \neq 0 \text{ at } (r,\theta).$$

1.4 Consider the graph of f as the graph of the parametric equations $x = r\cos\theta = f(\theta)\cos\theta, y = r\sin\theta = f(\theta)\sin\theta$.
$\dfrac{dy}{dx} =$

1.5 If the curve $r = f(\theta)$ passes through the origin at $\theta = \theta_0$, then $f(\theta_0) = 0$ and the slope equation gives

$$\left.\frac{dy}{dx}\right|_{(0,\theta_0)} = \underline{\qquad\qquad\qquad\qquad}$$

1.6 If the graph of $r = f(\theta)$ passes through the origin at $\theta = \theta_0$ the value the slope of the curve there is _____.

1.7 The reason we say _____ and not just _____ is that a polar curve may pass through the origin (or any point) _____, with different slopes at different.

Ex. 1 ..... (`example1, p632`)

Graph the curve $r = 1 - \cos\theta$.

sol:

✐ Ex. **2** ..... (`example2, p633`)

Graph the curve $r^2 = 4\cos\theta$.

sol:

✐ Ex. **3** ..... (`example3, p3`)722

Graph the curve $r^2 = \sin 2\theta$.

sol:

2 重要極座標圖形

2.1 圓 circle

$r = a\cos\theta$		$r = a\sin\theta$	
$a < 0$	$a > 0$	$a < 0$	$a > 0$

2.2 心臟線 Cardioid

$r = a(1 \pm \cos\theta)$		$r = a(1 \pm \sin\theta)$	
$a < 0$	$a > 0$	$a < 0$	$a > 0$

2.3 蚶線 LimaCon

$r = a \pm b\cos\theta$		$r = a \pm b\sin\theta$	
$a > b$	$a < b$	$a > b$	$a < b$

2.4 雙紐線 Lemniscates

$r^2 = a^2 \cos 2\theta$	$r^2 = a^2 \sin 2\theta$

2.5 玫瑰線 Rose curve

$r = a \cos n\theta$		$r = a \sin n\theta$	
$n : odd$	$n : even$	$n : odd$	$n : even$

實習課練習 (EXERCISE 11.4)

2. Graph the curve $r = 2 - 2\cos\theta$.

6. Graph the curve $r = 1 + 2\sin\theta$.

13. Graph the curve $r^2 = 4\cos 2\theta$.

19. Graph the curve $r = \sin 2\theta$.

THOMAS' CALCULUS (12/E)

11.5 Areas and Lengths in Polar Coordinates

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1 Area in the Plane

1.1 The fan-shaped circular sector has radius _____ and central angle of radian measure _____. The area is _____ times the area of a circle of radius _____, or

$$A_k = \underline{\qquad\qquad\qquad\qquad}, \quad \sum_{k=1}^{n} A_k = \underline{\qquad\qquad\qquad\qquad}$$

$$A = \underline{\qquad\qquad\qquad\qquad} = \underline{\qquad\qquad\qquad\qquad}$$

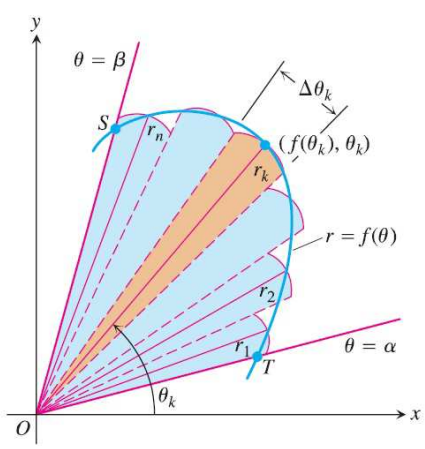

FIGURE 11.30 To derive a formula for the area of region *OTS*, we approximate the region with fan-shaped circular sectors.

1.2 *Area of the Fan-Shaped Region Between the Origin and the Curve* $r = f(\theta)$, $\alpha < \theta < \beta$

$$A = \underline{\qquad\qquad}.$$

This is the integral of the area differential $dA = \underline{\qquad\qquad} = \underline{\qquad\qquad}$.

1.3 *Area of the Region* $0 \le r_1(\theta) \le r \le r_2(\theta)$, $\alpha \le \theta \le \beta$

$$A = \underline{\qquad\qquad\qquad\qquad} = \underline{\qquad\qquad\qquad}$$

Ex. 1 ..... (`example1, p636`)

Find the area of the region in the plane enclosed by the cardioid $r = 2(1 + \cos\theta)$.

sol:

Ex. 2 ..... (example2, p636)

Find the area inside the smaller loop of the limacon $r = 2\cos\theta + 1$.

sol:

Ex. 3 ..... (example3, p637)

Find the area of the region that lies inside the circle $r = 1$ and outside the cardioid $r = 1 - \cos\theta$.

sol:

實習課練習 (EXERCISE 11.5)

3. Find the area of the region inside the oval limacon $r = 4 + 2\cos\theta$.

8. Find the area of the region inside the six-leaved rose $r^2 = 2\sin 3\theta$.

13. Find the area of the region inside the lemniscate $r^2 = 6\cos 2\theta$ and outside the circle $r = \sqrt{3}$.

14. Find the area of the region inside the circle $r = 3a\cos\theta$ and outside the cardioid $r = a(1 + \cos\theta),\ a > 0$.

THOMAS' CALCULUS (12/E)

12.2 Vectors

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1 Component Form

1.1 *Definition*

The vector represented by the _____ has initial point _____ and terminal point _____ and its length is denoted by _____. Two vectors are equal if they have the _____ and _____.

Definition

(a) If $\vec{v}$ is a two-dimensional vector in the plane equal to the vector with initial point at the _____ and terminal point _____, then the component form of _____ is _____.

(b) If $\vec{v}$ is a three-dimensional vector in the plane equal to the vector with initial point at the _____ and terminal point _____, then the component form of $\vec{v}$ is _____.

1.2 The magnitude or length of the vector $\vec{v} = \vec{PQ}$, $P(x_1, y_1, z_1), Q(x_2, y_2, z_2)$, is the nonnegative number

$$\|\vec{v}\| = \underline{\qquad\qquad} = \underline{\qquad\qquad\qquad\qquad}$$

Ex. 1 ..... (`example1, p666`)

Find the component form and length of the vector with initial point $P(-3, 4, 1)$ and terminal $Q(-5, 2, 2)$.

sol:

2 Vector Algebra Operations

2.1 *Definition*

Let $\vec{u} =< u_1, u_2, u_3 >$ and $\vec{v} =< v_1, v_2, v_3 >$ be vectors with k a scalar.

(a) Addition: $\vec{u} + \vec{v} =$ _____

(b) Scalar multiplication: $k\vec{u} =$ _____

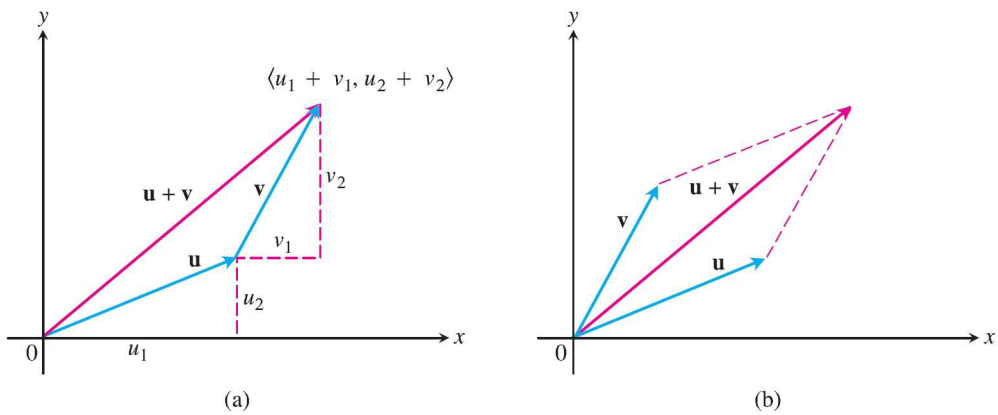

FIGURE 12.12 (a) Geometric interpretation of the vector sum. (b) The parallelogram law of vector addition.

2.2 *Properties of Vector Operations*

Let $\vec{u}, \vec{v}, \vec{w}$ be vectors and a, b be scalars.

1. $\vec{u} + \vec{v} =$ _____ 2. $(\vec{u} + \vec{v}) + \vec{w} =$ _____
3. $\vec{u} + \vec{0} =$ _____ 4. $\vec{u} + (-\vec{u}) =$ _____
5. $0\vec{u} = \vec{0}$ 6. $1\vec{u} = \vec{u}$
7. $a(b\vec{u}) =$ _____ 8. $a(\vec{u} + \vec{v}) =$ _____
9. $(a + b)\vec{u} =$ _____

2.3 A vector $\vec{v}$ of length 1 is called _____.

2.4 The standard unit vector are _____, _____, and _____.

2.5 Any vector $\vec{v} =< v_1, v_2, v_3 >$ can be written as a linear combination of the standard

unit:

$$\begin{aligned}\vec{v} &= \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}\\ &= \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}\\ &= \underline{\qquad\qquad\qquad\qquad}\end{aligned}$$

2.6 The scalar _____ is the _____ (j-component, k-component) of the vector $\vec{v}$.

2.7 The vector from $P(x_1, y_1, z_1)$ to $Q(x2, y_2, z_2)$ is

$$\vec{PQ} = \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

2.8 Whenever $\vec{u} \neq \vec{0}$, _____ is a unit vector in the direction of $\vec{v}$.

2.9 The equation $\vec{v} =$ _____ expresses $\vec{v}$ as its length times its direction.

2.10 The midpoint M of the line segment joining points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is the point _____.

Ex. 2 ..... (example3, p668)

Let $\vec{u} =< -1, 3, 1 >$ and $\vec{v} =< 4, 7, 0 >$. Find the component of (a) $2\vec{u} + 3\vec{v}$ (b) $\vec{u} - \vec{v}$ (c) $\|\frac{1}{2}\vec{u}\|$.

sol:

Ex. 3 ..... (example4, p669)

Find a unit vector $\vec{u}$ in the direction of the vector from $P_1(1, 0, 1)$ to $P_2(3, 2, 0)$.

sol:

實習課練習 (EXERCISE 12.2)

☐ In Exercise 17-22, express each vector in the form $\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$.

18. $\overrightarrow{P_1P_2}$ if $\vec{P_1}$ is the point $(1, 2, 0)$ and P_2 is the point $(-3, 0, 5)$.

22. $-2\vec{u} + 3\vec{v}$ if $\vec{u} =< -1, 0, 2 >$ and $\vec{v} =< 1, 1, 1 >$.

25. Express $2\vec{i} + \vec{j} - 2\vec{k}$ as a product of its length and direction.

33. Find a vector of magnitude 7 in the direction of $\vec{v} = 12\vec{i} - 5\vec{k}$.

THOMAS' CALCULUS (12/E)

12.3 The Dot Product

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1 Angle Between Vectors

1.1 *Definition*

The dot product $\vec{u} \cdot \vec{v}$ of vectors $\vec{u} =< u_1, u_2, u_3 >$ and $\vec{v} =< v_1, v_2, v_3 >$ is

$$\vec{u} \cdot \vec{v} = \underline{\qquad\qquad\qquad\qquad}$$

1.2 *Theorem 1*

The angle θ between two nonzero vectors $\vec{u} =< u_1, u_2, u_3 >$ and $\vec{v} =< v_1, v_2, v_3 >$ is given by

$$\theta = \underline{\qquad\qquad\qquad\qquad\qquad\qquad} = \underline{\qquad\qquad\qquad\qquad}$$

✐ Ex. 1 ..... (`example2, p676`)

Find the angle between $\vec{u} = \vec{i} - 2\vec{j} - 2\vec{k}$ and $\vec{v} = 6\vec{i} + 3\vec{j} + 2\vec{k}$.

sol:

✎ Ex. **2** ..... (`example3, p676`)

Find the angle θ in the triangle ABC determined by the vertices $A = (0, 0)$, $B = (3, 5)$ and $C = (5, 2)$.

sol:

2 Perpendicular (Orthogonal) Vectors

2.1 *Definition*

Vectors $\vec{u}$ and $\vec{v}$ are _____ (or _____) if and only if _____.

2.2 *Properties of the Dot Product*

If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are any vectors and c is a scalar, then

1. $\vec{u} \cdot \vec{v} =$ _____
2. $(c\vec{u}) \cdot \vec{v} =$ _____ $=$ _____
3. $\vec{u} \cdot (\vec{v} + \vec{w}) =$ _____
4. $\vec{u} \cdot \vec{u} =$ _____
5. $\vec{0} \cdot \vec{u} =$ _____

2.3 The vector projection of $\vec{u} = \vec{PQ}$ onto a nonzero vector $\vec{v} = \vec{PS}$ is the vector $\vec{PR}$ determined by dropping a perpendicular from Q to the line PS:

_____

2.4 The scalar component of $\vec{u}$ in the direction $\vec{v}$ is the scalar

_____ $=$ _____ $=$ _____

Ex. 3 ..... (example5, p678)

Find the vector projection of $\vec{u} = 6\vec{i} + 3\vec{j} + 2\vec{k}$ onto $\vec{v} = \vec{i} - 2\vec{j} - 2\vec{k}$ and the scalar component of $\vec{u}$ in the direction of $\vec{v}$.

sol:

實習課練習 (EXERCISE 12.3)

☐ Find

(a) $\vec{v} \cdot \vec{u}$, $\|\vec{v}\|$, $\|\vec{u}\|$

(b) the cosine of the angle between $\vec{v}$ and $\vec{u}$.

(c) the scalar component of $\vec{u}$ in the direction of $\vec{v}$

(d) the vector $\text{proj}_{\vec{v}}\vec{u}$

4. $\vec{v} = 2\vec{i} + 10\vec{j} - 11\vec{k}$, $\vec{u} = 2\vec{i} + 2\vec{j} + \vec{k}$

8. $\vec{v} =< \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}} >$, $\vec{u} =< \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{3}} >$

THOMAS' CALCULUS (12/E)

12.4 The Cross Product

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1 The Cross Product of Two Vectors in Space

1.1 *Definition*

Let the unit vector $\vec{n}$ be perpendicular to the plane by the right-hand rule. Then the _____ _____ ($\vec{u}$ cross $\vec{v}$) is the vector defined by

$$\vec{u} \times \vec{v} = \underline{\qquad\qquad}$$

1.2 Nonzero vectors $\vec{u}$ and $\vec{v}$ are _____ if and only if _____.

1.3 *Properties of the Cross Product*

If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are any vectors and r, s are scalars, then

1. $(r\vec{u}) \times (s\vec{v}) =$ _____
2. $\vec{u} \times (\vec{v} + \vec{w}) =$ _____
3. $\vec{v} \times \vec{u} =$ _____
4. $(\vec{v} + \vec{w}) \times \vec{u} =$ _____
5. $\vec{0} \times \vec{u} =$ _____
6. $\vec{u} \times (\vec{v} \times \vec{w}) =$ _____

1.4 (a) $\vec{i} \times \vec{j} =$ _____

(b) $\vec{j} \times \vec{k} =$ _____

(c) $\vec{k} \times \vec{i} =$ _____

(d) $\vec{i} \times \vec{i} =$ _____

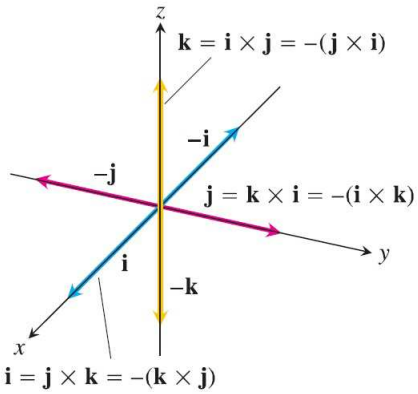

FIGURE 12.29 The pairwise cross products of **i**, **j**, and **k**.

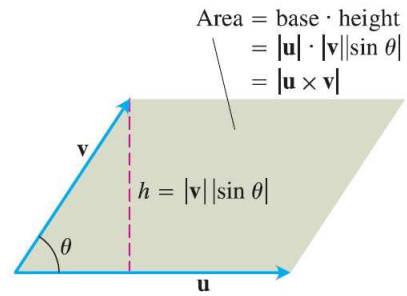

FIGURE 12.30 The parallelogram determined by **u** and **v**.

1.5 If $\vec{u} = u_1\vec{i} + u_2\vec{j} + u_3\vec{k}$ and $\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$, then

$$\vec{u} \times \vec{v} =$$

pf.

1.6 Calculating the triple scalar product as a determinnat.

$$(\vec{u} \times \vec{v}) \cdot \vec{w} =$$

pf.

1.7 The magnitude of $\vec{u} \times \vec{v}$ is the area of the _____ determined by $\vec{u}$ and $\vec{v}$, $\vec{u}$ being the _____ of the parallelogram and $\|\vec{v}\| \|\sin\theta\|$ the _____.

$$\|\vec{u} \times \vec{v}\| = \underline{\qquad\qquad} = \underline{\qquad\qquad}$$

✐ Ex. **1** ..... (`example1, p684`)

Find $\vec{u} \times \vec{v}$ and $\vec{v} \times \vec{u}$ if $\vec{u} = 2\vec{i} + \vec{j} + \vec{k}$ and $\vec{v} = -4\vec{i} + 3\vec{j} + \vec{k}$.

sol:

✐ Ex. **2** ..... (`example2, p684`)

Find a vector perpendicular to the plane of $P(1, -1, 0)$, $Q(2, 1, -1)$, and $R(-1, 1, 2)$.

sol:

✐ Ex. **3** ..... (`example4, p684`)

Find a unit vector perpendicular to the plane of $P(1, -1, 0)$, $Q(2, 1, -1)$, and $R(-1, 1, 2)$.

sol:

實習課練習 (EXERCISE 12.4)

☐ In Exercise 1-8, find the length and direction of $\vec{u} \times \vec{v}$ and $\vec{v} \times \vec{u}$.

3. $\vec{u} = 2\vec{i} - 2\vec{j} + 4\vec{k}$, $\vec{v} = -\vec{i} + \vec{j} - 2\vec{k}$

6. $\vec{u} = \vec{i} \times \vec{j}$, $\vec{v} = \vec{j} \times \vec{k}$

☐ In Exercise 15-18, (a) Find the area of the triangle determined by the points P, Q and R. (b) Find a unit vector perpendicular to plane PQR.

16. $P(1, 1, 1)$, $Q(2, 1, 3)$, $R(3, -1, 1)$.

THOMAS' CALCULUS (12/E)

14.2 Limits and Continuity in Higher Dimensions

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1 Limits for Functions of Two Variables

1.1 *Definition*

We say that a function _____ approaches the limit _____ as _____ approaches _____, and write

_____

if, for every number $\epsilon > 0$, there exists a corresponding number $\delta > 0$ such that for all (x, y) in the domain of f,

_____ whenever _____.

1.2 *Theorem 1: Properties of Limits of Functions of Two Variables*

The following rules hold if L, M and k are real numbers and

$$\lim_{(x,y)\to(x_0),y_0)} f(x,y) = _____ \quad \text{and} \quad \lim_{(x,y)\to(x_0),y_0)} g(x,y) = _____$$

(a) *Sum Rule:* $\lim_{(x,y)\to(x_0),y_0)} (f(x,y) + g(x,y)) =$ _____

(b) *Difference Rule:* $\lim_{(x,y)\to(x_0),y_0)} (f(x,y) - g(x,y)) =$ _____

(c) *Constant Multiple Rule:* $\lim_{(x,y)\to(x_0),y_0)} kf(x,y) =$ _____

(d) *Product Rule:* $\lim_{(x,y)\to(x_0),y_0)} (f(x,y) \cdot g(x,y)) =$ _____

(e) *Quotient Rule:* $\lim\limits_{(x,y)\to(x_0),y_0)} \dfrac{f(x,y)}{g(x,y))} =$ _____, $M \neq 0$.

(f) *Power Rule:* $\lim\limits_{(x,y)\to(x_0),y_0)} [f(x,y)]^n =$ _____, $n \in N^+$.

(g) *Root Rule:* $\lim\limits_{(x,y)\to(x_0),y_0)} \sqrt[n]{f(x,y)} =$ _____, $n \in N^+$.

✐ Ex. **1** ..... (`example1, p757`)

(a) $\lim\limits_{(x,y)\to(0,1)} \dfrac{x - xy + 3}{x^2y + 5xy - y^3} =$

(b) $\lim\limits_{(x,y)\to(3,-4)} \sqrt{x^2 + y^2} =$

✐ Ex. **2** ..... (`example2, p757`)

Find $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$.

sol:

2 Continuity

2.1 *Definition*

A function $f(x,y)$ is _____ at the point (x_0, y_0) if

(a) f is _____ at (x_0, y_0),

(b) _____ exists,

(c) _____.

A function is _____ if it is continuous at every point of its domain.

2.2 *Two-Path Test for Nonexistence of a Limit*

If a function $f(x, y)$ has a different limits along _____ in the domain of f as (x, y) approaches (x_0, y_0), then $\lim_{(x,y)\to(x_0,y_0)} f(x, y)$ does not exist.

實習課練習 (EXERCISE 14.2)

2. $\displaystyle\lim_{(x,y)\to(0,4)} \frac{x}{\sqrt{y}}$

3. $\displaystyle\lim_{(x,y)\to(3,4)} \sqrt{x^2+y^2-1}$

7. $\displaystyle\lim_{(x,y)\to(0,\ln 2)} e^{x-y}$

12. $\displaystyle\lim_{(x,y)\to(\pi/2,0)} \frac{\cos y+1}{y-\sin x}$

14. $\displaystyle\lim_{\substack{(x,y)\to(1,1)\\ x\neq y}} \frac{x^2-y^2}{x-y}$

18. $\displaystyle\lim_{\substack{(x,y)\to(2,2)\\ x+y\neq 4}} \frac{x+y-4}{\sqrt{x+y}-2}$

21. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2}$

22. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{1-\cos xy}{xy}$

THOMAS' CALCULUS (12/E)

14.3 Partial Derivatives

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1 Partial Derivatives of a Function of Two Variables

1.1 *Definition*

The _____ of $f(x, y)$ with respect to x at the point (x_0, y_0) is

$$\underline{\qquad\qquad} = \underline{\qquad\qquad\qquad\qquad}$$

provided the limit exists.

1.2 An equivalent expression

_____

1.3 Several notations

_____, _____, _____, _____, _____

1.4 The partial derivative of $f(x, y)$ with respect to _____ is obtained by differentiating f in the usual way while treating _____ as a _____.

✐ Ex. 1 ..... (example1, p766)

Find the values of $\partial f/\partial x$ and $\partial f/\partial y$ at the point $(4, -5)$ if $f(x, y) = x^2 + 3xy + y - 1$.

sol:

Ex. 2 ..... (example2, p767)

Find $\partial f/\partial y$ as a function if $f(x, y) = y \sin xy$.

sol:

Ex. 3 ..... (example3, p767)

Find f_x and f_y as functions if $f(x, y) = \dfrac{2y}{y + \cos x}$.

sol:

Ex. 4 ..... (example4, p767)

Find $\partial z/\partial x$ if the equation $yz - \ln z = x + y$ defines z as a function of the two independent variables x and y and the partial derivative exists.

sol:

✐ Ex. 5 ..... (example6, p768)

Find $\partial f/\partial z$ if x, y and z are independent variables and $f(x, y, z) = x\sin(y + 3z)$.

sol:

2 Second-Order Partial Derivatives

2.1 Differentiate $f(x, y)$ twice, we produce its second-order derivatives.

_____, _____, _____, _____

2.2 *Theorem 2: The Mixed Derivative Theorem*

> If $f(x, y)$ and its partial derivatives _____ are defined throughout an open region containing a point (a, b) and are all continuous at (a, b), then _____.

✐ Ex. 6 ..... (example9, p770)

If $f(x, y) = x\cos y + ye^x$, find the second-order derivatives.

sol:

✐ Ex. 7 ..... (example10, p770)

Find $\partial^2 w/\partial x \partial y$ if $w = xy + \dfrac{e^y}{y^2+1}$.

sol:

✐ Ex. 8 ..... (example11, p771)

Find f_{yxyz} if $f(x, y, z) = 1 - 2xy^2z + x^2y$.

sol:

實習課練習 (EXERCISE 14.3)

7. Find $\partial f/\partial x$ and $\partial f/\partial y$: $f(x,y) = \sqrt{x^2+y^2}$.

16. Find $\partial f/\partial x$ and $\partial f/\partial y$: $f(x,y) = e^{xy}\ln y$.

21. Find $\partial f/\partial x$ and $\partial f/\partial y$: $f(x,y) = \displaystyle\int_x^y g(t)\ dt$, ($g$ is continuous for all t).

22. Find $\partial f/\partial x$ and $\partial f/\partial y$: $f(x,y) = \displaystyle\sum_{n=1}^{\infty}(xy)^n$.

25. Find f_x, f_y and f_z: $f(x,y,z) = x - \sqrt{y^2+z^2}$.

32. Find f_x, f_y and f_z: $f(x,y,z) = e^{-xyz}$.

43. Find all the second-order partial derivatives: $g(x,y) = x^y + \cos y + y\sin x$.

52. Verify that $w_{xy} = w_{yx}$: $w = e^x + x\ln y + y\ln x$.

66. Find the value of $\partial x/\partial z$ at the point $(1,-1,-3)$ if the equation $xz+y\ln x-x^2+4=0$ defines x as a function of the two independent variables y and z and the partial dervitive exists.

72. Let $f(x,y) = \begin{cases} \sqrt{x}, & x \geq 0 \\ x^2, & x < 0. \end{cases}$

Find f_x, f_y, f_{xy} and f_{yx} and state the domain for each partial derivative.

THOMAS' CALCULUS (12/E)

14.4 The Chain Rule

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1 The Chain Rule

1.1 *Theorem 5: Chain Rule for Functions of Two Independent Variables*

If _____ is differentiable and if _____, _____ are differentiable functions of t, then the composite _____ is a differentiable function of t and

$$\frac{dw}{dt} = \underline{\qquad\qquad\qquad}$$

1.2 *Theorem 6: Chain Rule for Functions of Three Independent Variables*

If $w = f(x, y, z)$ is differentiable and x, y, and z are differentiable functions of t, then the composite $w = f(x(t), y(t), z(t))$ is a differentiable function of t and

$$\frac{dw}{dt} = \underline{\qquad\qquad\qquad\qquad\qquad}$$

1.3 The Branch Diagram:

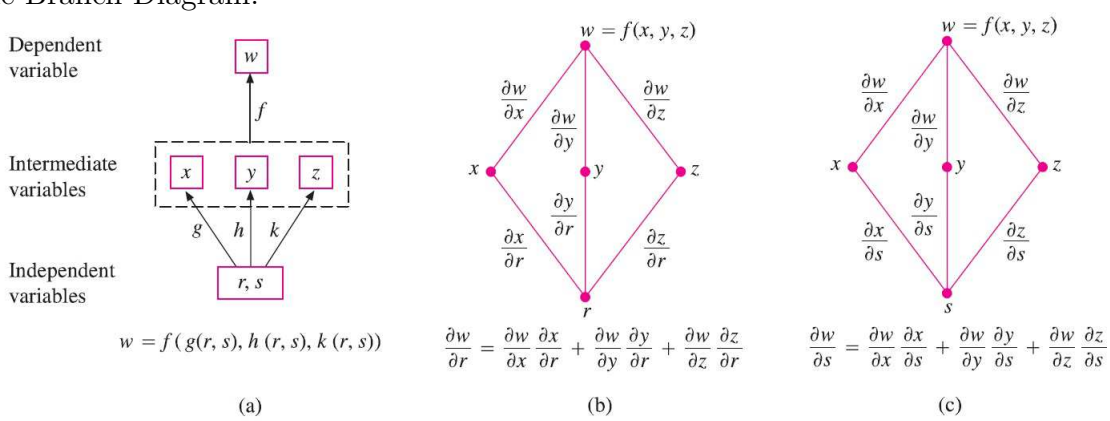

FIGURE 14.21 Composite function and branch diagrams for Theorem 7.

1.4 *Theorem 7: Chain Rule for Two Independent Variables and Three Intermediate Variables*

Suppose that $w = f(x, y, z)$, _____, _____ and _____. If all four functions are differentiable, then w has partial derivatives with respect to r and s, given by

$$\frac{dw}{dr} = \underline{\hspace{6cm}}$$

$$\frac{dw}{ds} = \underline{\hspace{6cm}}$$

Ex. 1 ..... (`example1, p776`)

Use the Chain Rule to find the derivative of $w = xy$ with respect to t along the path $x = \cos t$, $y = \sin t$. What is the derivative's value at $t = \pi/2$.

sol:

Ex. 2 ..... (`example2, p777`)

Find dw/dt if $w = xy + z$, $x = \cos t$, $y = \sin t$, $z = t$.

sol:

✐ Ex. **3** ..... (`example3, p778`)

Express $\partial w/\partial r$ and $\partial w/\partial s$ interm of r and s if $w = x+2y+z^2$, $x = \dfrac{r}{s}$, $y = r^2+\ln s$, $z = 2r$.

sol:

✐ Ex. **4** ..... (`example3, p779`)

Express $\partial w/\partial r$ and $\partial w/\partial s$ interm of r and s if $w = x^2 + y^2$, $x = r - s$, $y = r + s$.

sol:

2 Implicit Differentiation

2.1 *Theorem 8: A Formula for Implicit Differentiation*

Suppose that $F(x, y)$ is differentiable and that the equation $F(x, y) = 0$ defines y as a differentiable function of x. Then at any point where $F_y \neq 0$,

$$\frac{dy}{dx} = \underline{\qquad\qquad}$$

pf.

2.2 Suppose that $F(x, y, z) = 0$ defines z implicitly as a function $z = f(x, y)$:

$$\frac{\partial z}{\partial x} = \underline{\qquad\qquad}, \quad \frac{\partial z}{\partial y} = \underline{\qquad\qquad}$$

Ex. **5** ..... (`example5, p780`)

Use implicit differentiation to find dy/dx if $y^2 - x^2 - \sin xy = 0$.

sol:

Ex. **6** ..... (`example6, p781`)

Find $\partial z/\partial x$ and $\partial z/\partial y$ at (0,0,0) if $x^3 + z^2 + ye^{xz} + z\cos y = 0$.

sol:

實習課練習 (EXERCISE 14.4)

3. Evaluate dw/dt at the given value of t: $w = \dfrac{x}{z} + \dfrac{y}{z}$, $x = \cos^2 t$, $y = \sin^2 t$, $z = 1/t$, $t = 3$.

5. Evaluate dw/dt at the given value of t: $w = 2ye^x - \ln z$, $x = \ln(t^2+1)$, $y = \tan^{-1} t$, $z = e^t$, $t = 1$.

8. Evaluate $\partial z/\partial u$ and $\partial z/\partial v$ at the given point (u, v): $z = \tan^{-1}(x/y)$, $x = u\cos v$, $y = u\sin v$, $(u, v) = (1.3, \pi/6)$.

9. Evaluate $\partial w/\partial u$ and $\partial w/\partial v$ at the given point (u, v): $w = xy + yz + xz$, $x = u + v$, $y = u - v$, $z = uv$, $(u, v) = (1/2, 1)$.

11. Evaluate $\partial u/\partial x$, $\partial u/\partial y$ and $\partial u/\partial z$ at the given point (x, y, z): $u = \dfrac{p-q}{q-r}$, $p = xy + yz + xz$, $q = x - y + z$, $r = x + y - z$, $(x, y, z) = (\sqrt{3}, 2, 1)$.

21. Draw a branch diagram and write a Chain Rule formula for each derivative. $\partial w/\partial s$ and $\partial w/\partial t$ for $w = g(u)$, $u = h(s, t)$.

28. Find dy/dx at the given point: $xe^y + \sin xy + y - \ln 2 = 0$, $(0, \ln 2)$

29. Find the values of $\partial z/\partial x$ and $\partial z/\partial y$ at the given point: $z^3 - xy + yz + y^3 - 2 = 0$, $(1, 1, 1)$

35. Find $\partial w/\partial v$ when $u = 0$, $v = 0$ if $w = x^2 + (y/x)$, $x = u - 2v + 1$, $y = 2u + v - 2$.

THOMAS' CALCULUS (12/E)

14.5 Directional Derivatives and Gradient Vectors

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1 Directional Derivatives in the Plane

1.1 Suppose that the function _____ is defined throughout a region R in the xy-plane, that _____ is a point in R, and that _____ is a unit vector. Then the equations

$$x = _____, \quad y = _____$$

parametrize the line through P_0 parallel to $\vec{u}$.

1.2 If the parameter s measures _____ from P_0 in the direction of _____, we find the rate of change of f at P_0 in the direction of $\vec{u}$ by calculating _____ at _____.

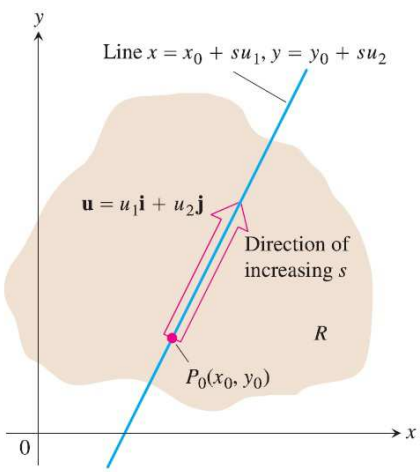

FIGURE 14.26 The rate of change of f in the direction of $\mathbf{u}$ at a point P_0 is the rate at which f changes along this line at P_0.

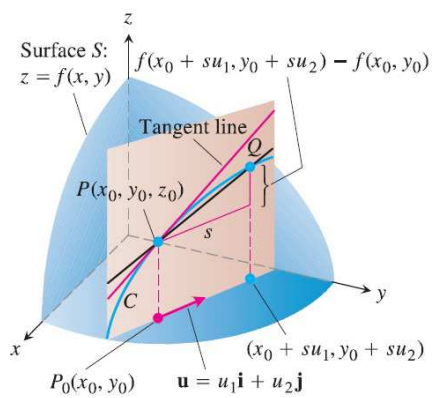

FIGURE 14.27 The slope of curve C at P_0 is $\lim_{Q \to P}$ slope (PQ); this is the directional derivative

$$\left(\frac{df}{ds}\right)_{\mathbf{u}, P_0} = (D_{\mathbf{u}} f)_{P_0}.$$

1.3 *Definition*

> The derivative of f at $P_0(x_0, y_0)$ in the direction of the unit vector $\vec{u} = u_1\vec{i} + u_2\vec{j}$ is the number
>
> $$\underline{\qquad\qquad} = \underline{\qquad\qquad\qquad\qquad\qquad\qquad}$$
>
> povided the limit exists.

1.4 The derivative of f at P_0 in the direction of $\vec{u}$ is also defined by _____.

1.5 The partial defivatives $f_x(x_0, y_0)$ and $f_y(x_0, y_0)$ are the _____ of f at P_0 in the _____ and _____ directions.

Ex. 1 ..... (`example1, p785`)

Using the definition, find the derivative of $f(x, y) = x^2 + xy$ at $P_0(1, 2)$ in the direction of the unit vector $\vec{u} = (1/\sqrt{2})\vec{i} + (1/\sqrt{2})\vec{j}$

sol:

2 Calculation and Gradients

2.1 *Definition*

> The gradient vector (_____) of $f(x, y)$ at a point $P_0(x_0, y_0)$ is the vector
>
> $$\underline{\qquad\qquad} = \underline{\qquad\qquad\qquad\qquad}$$

2.2 *Theorem 9: The Directional Derivative Is a Dot Product*

If $f(x, y)$ is differentiable in an open region containing $P_0(x_0, y_0)$, then

_____

the dot product of _____ and _____.

✐ Ex. **2** ..... (`example2, p786`)

Find the derivative of $f(x, y) = xe^y + \cos(xy)$ at the point $(2, 0)$ in the direction of $\vec{v} = 3\vec{i} - 4\vec{j}$.

sol:

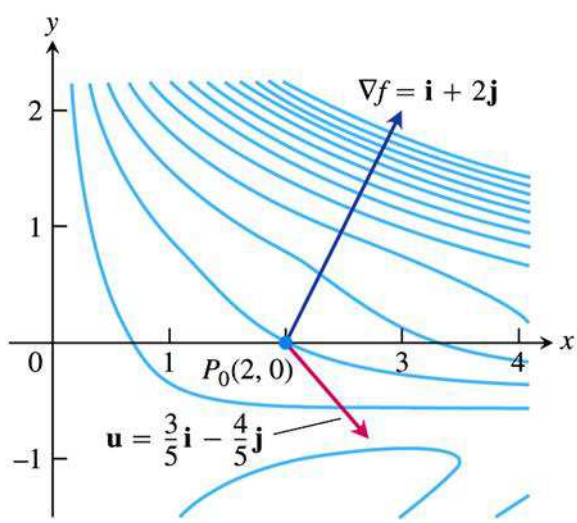

FIGURE 14.28 Picture ∇f as a vector in the domain of f. The figure shows a number of level curves of f. The rate at which f changes at (2, 0) in the direction $\mathbf{u} = (3/5)\mathbf{i} - (4/5)\mathbf{j}$ is $\nabla f \cdot \mathbf{u} = -1$ (Example 2).

實習課練習 (EXERCISE 14.5)

5. Find the gradient of the function at the given point. $f(x,y) = \sqrt{2x+3y}$, $(-1,2)$

9. Find ∇f at the given point: $f(x,y,z) = e^{x+y}\cos z + (y+1)\sin^{-1} x$, $(0,0,\pi/6)$.

☐ In Exercise 11-18, find the derivative of the function at P_0 in the direction of $\vec{u}$.

11. $f(x,y) = 2xy - 3y^2$, $P_0(5,5)$, $\vec{u} = 4\vec{i} + 3\vec{j}$.

13. $g(x,y) = \dfrac{x-y}{xy+2}$, $P_0(1,-1)$, $\vec{u} = 12\vec{i} + 5\vec{j}$.

15. $f(x,y,z) = xy + yz + zx$, $P_0(1,-1,2)$, $\vec{u} = 3\vec{i} + 6\vec{j} - 2\vec{k}$.

17. $g(x,y,z) = 3e^x \cos yz$, $P_0(0,0,0)$, $\vec{u} = 2\vec{i} + \vec{j} - 2\vec{k}$.

THOMAS' CALCULUS (12/E)

15.1 Double and Iterated Integrals over Rectangles

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1 Double Integrals

1.1 A rectangle with sides parallel to the coordinate axes: _____.

1.2 *Definition: The Double Integrals*

> Let f be a function of two variables that is defined on a _____.
> If
>
> _____
>
> exists, we say that f is _____ on R. Moreover, _____, called the double integral of f over R, is then given by
>
> $$\iint_R f(x,y)\,dA = \underline{\qquad\qquad\qquad\qquad}$$

1.3 If $f(x) \geq 0$, _____ represents the _____ under the curve $y = f(x)$ between a and b.

1.4 If $f(x,y) \geq 0$, _____ represents the _____ under the surface $z = f(x,y)$ and above the rectangle R.

1.5 *Theorem: Fubini's Theorem (First Form)*

If $f(x, y)$ is continuous throughout the rectangular region $R : a \leq\leq b, c \leq y \leq d$, then

$$V = \underline{\qquad\qquad} = \underline{\qquad\qquad} = \underline{\qquad\qquad}$$

$$\underline{\qquad\qquad\qquad} = \underline{\qquad\qquad\qquad}.$$

The expression of the right, called an iterated or repeated integral.

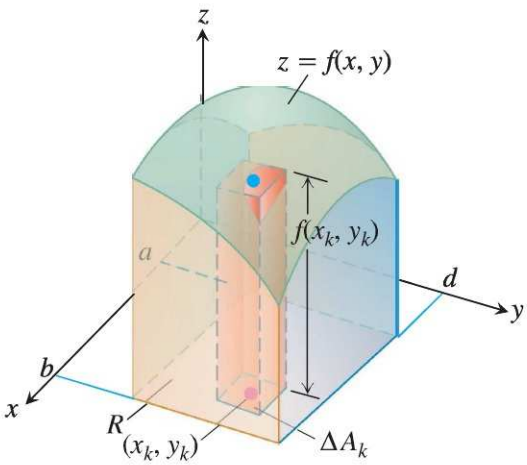
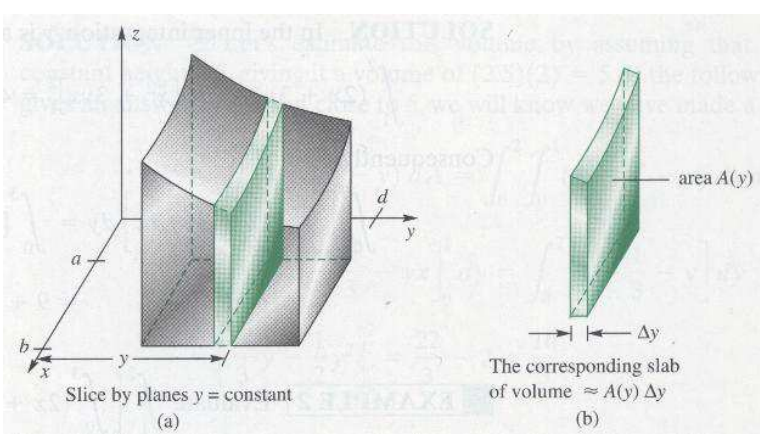
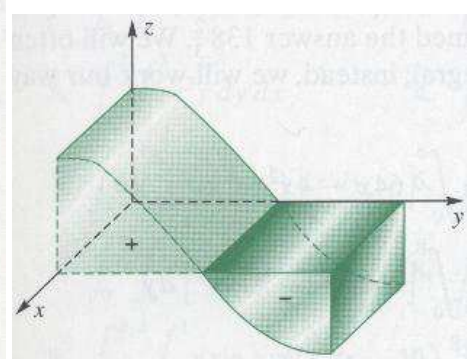


FIGURE 15.2 Approximating solids with rectangular boxes leads us to define the volumes of more general solids as double integrals. The volume of the solid shown here is the double integral of $f(x, y)$ over the base region R.

1.6 If $f(x, y)$ is $\underline{\qquad\qquad}$ on part of R, then $\iint\limits_R f(x, y)\, dA$ gives the $\underline{\qquad\qquad\qquad}$ of the solid between the surface $z = f(x, y)$ and the rectangle R of the $xy-$plane. The actual volume of this slid is $\qquad\qquad$.

$$\underline{\qquad\qquad\qquad}$$





✐ Ex. **1** ..... (`example1, p839`)

Calculate $\iint\limits_R f(x,y)\ dA$ for $f(x,y) = 100 - 6x^2y$ and $R : 0 \le x \le 2,\ -1 \le y \le 1$.

sol:

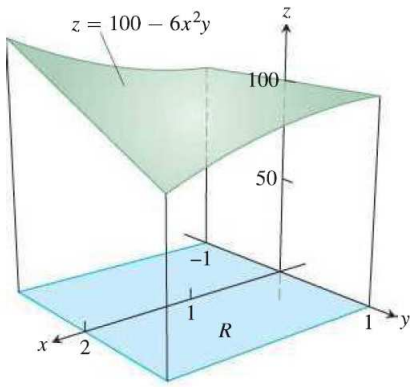


FIGURE 15.6 The double integral $\iint_R f(x, y)\ dA$ gives the volume under this surface over the rectangular region R (Example 1).

✐ Ex. **2** ..... (`example2, p840`)

Find the volume of the region bounded above by the elliptical paraboloid $z = 10 + x^2 + 3y^2$ and below by the rectangle $R : 0 \le x \le 1, 0 \le y \le 2$.

sol:

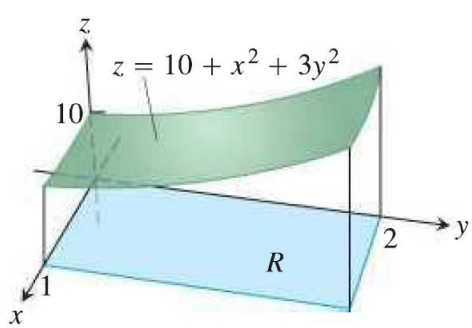


FIGURE 15.7 The double integral $\iint_R f(x, y)\ dA$ gives the volume under this surface over the rectangular region R (Example 2).

實習課練習 (EXERCISE 15.1)

4. $\displaystyle\int_0^1 \int_0^1 (1 - \frac{x^2+y^2}{2})\ dx\ dy$

7. $\displaystyle\int_0^1 \int_0^1 \frac{y}{1+xy}\ dx\ dy$

11. $\displaystyle\int_{-1}^2 \int_0^{\pi/2} y \sin x\ dx\ dy$

14. $\displaystyle\iint_R (\frac{\sqrt{x}}{y^2}) dA, \quad R: 0 \le x \le 4, 1 \le y \le 2.$

17. $\displaystyle\iint_R e^{x-y}, \quad R: 0 \le x \le \ln 2, 0 \le y \le 2.$

25. Find the volume of the region bounded above by the plane $z = 2 - x - y$ and below by the square $R: 0 \le x \le 1, 0 \le y \le 1$.

THOMAS' CALCULUS (12/E)

15.2 Double Integrals over General Regions

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1 Double Integrals over Bounded, Nonrectangular Regions

1.1 *Theorem: Fubini's Theorem (Stronger Form)*

Let $f(x, y)$ be continuous on a region R.

(a) If R is defined by $a \le x \le b, g_1(x) \le y \le g_2(x)$, with g_1 and g_2 continuous on $[a, b]$, then

$$\iint_R f(x, y)\ dA = \underline{\qquad\qquad\qquad\qquad}.$$

(b) If R is defined by $c \le y \le d, h_1(y) \le x \le h_2(y)$, with h_1 and h_2 continuous on $[c, d]$, then

$$\iint_R f(x, y)\ dA = \underline{\qquad\qquad\qquad\qquad}.$$

✐ Ex. 1 ..... (`example1, p843`)

Find the volume of the prism whose base is the triangle in the xy-plane bounded by hte x-axis and the line $y = x$ and $x = 1$ and whose top lies in the plane $z = f(x, y) = 3 - x - y$.

sol:

✎ Ex. **2** ..... (`example2, p844`)

Calculate $\iint\limits_R \frac{\sin x}{x}\, dA$ where R is the triangle in the xy-plane bounded by the x-axis, the line $y = x$, and the line $x = 1$.

sol:

2 Finding Limits of Integration

2.1 *Using Vertical Cross-sections*

When faced with evaluating $\iint_R f(x, y)\, dA$, integrating first with respect to y and then with respect to x, do the following:

(a) Sketch the _____ and label the _____.

(b) Find the _____ of integration. (c) Find the _____ of integration.

2.2 *Using Horizontal Cross-sections*

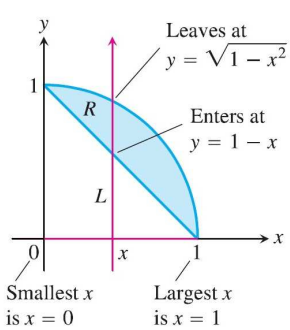

FIGURE 15.14 Finding the limits of integration when integrating first with respect to y and then with respect to x.

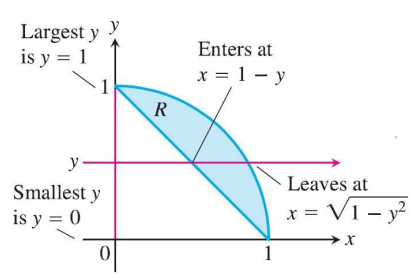

FIGURE 15.15 Finding the limits of integration when integrating first with respect to x and then with respect to y.

2.3 *Properties of Double Integrals*

If $f(x, y)$ and $g(x.y)$ are continuous on the bouneded region R, then

(a) Constant multiple: $\iint\limits_R kf(x, y)\ dA =$ _____ .

(b) Sum and difference: $\iint\limits_R [f(x, y) \pm g(x, y)]\ dA =$ _____ .

(c) Additivity: $\iint\limits_R f(x, y)\ dA =$ _____ .

(d) Domination: If $f(x, y) \geq g(x, y)$, then _____ .

✐ Ex. **3** ..... (`example3, p846`)

$$\int_0^2 \int_{x^2}^{2x} (4x + 2)\ dydx$$

sol:

✐ Ex. **4** ..... (`example4, p847`)

Find the volume of the wedgelike solid that lies beneath the surface $z = 16 - x^2 - y^2$ and above the region R bounded by the curve $y = 2\sqrt{x}$, the line $y = 4x - 2$, and the x-axis.

sol:

實習課練習 (EXERCISE 15.2)

19. $\displaystyle\int_0^{\pi}\int_0^{x} x\sin y\ dydx.$

21. $\displaystyle\int_1^{\ln 8}\int_0^{\ln y} e^{x+y}\ dxdy.$

25. Integrate f over the given region: $f(x,y)=x/y$ over the region in the first quadrant bounded by the lines $y=x$, $y=2x$, $x=1$ and $x=2$.

36. $\displaystyle\int_0^{1}\int_{1-x}^{1-x^2} dydx.$

43. $\displaystyle\int_1^{e}\int_0^{\ln x} xy\ dydx.$

44. $\displaystyle\int_0^{\pi/6}\int_{\sin x}^{1/2} xy^2\ dydx.$

47. $\displaystyle\int_0^{\pi}\int_x^{\pi} \frac{\sin y}{y}\ dydx.$

49. $\displaystyle\int_0^{1}\int_y^{1} x^2e^{xy}\ dxdy.$

60. Find the volume of the solid in the first octant bounded by the coordinate planes, the cylinder $x^2+y^2=4$, and the plane $z+y=3$.

THOMAS' CALCULUS (12/E)

15.3 Area by Double Integration

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1 Areas of Bounded Regions in the Plane

1.1 If we take $f(x, y) = 1$ in the definition of the double integral over a region R, the Riemann sums reduce to

$$S_n = \underline{\qquad\qquad\qquad\qquad} = \underline{\qquad\qquad}.$$

This is the sum of areas of all small rectangles in the partition of R.

1.2 *Definition*

> The area of a closed, bounded plane region R is
> $$A = \underline{\qquad\qquad\qquad\qquad}$$

1.3 The average value of f over R:

✐ Ex. 1 ..... (example1, p850)

Find the area of the region R bounded by $y = x$ and $y = x^2$ in the first quadrant.

sol:

Ex. 2 ..... (example2, p851)

Find the area of the region R bounded by $y = x^2$ and the line $y = x + 2$.

sol:

Ex. 3 ..... (example3, p852)

Find the average value of $f(x, y) = x \cos xy$ over the rectangle $R : 0 \leq x \leq \pi, 0 \leq y \leq 1$.

sol:

實習課練習 (EXERCISE 15.3)

☐ In Exercise 1-12, express the region's area as an iterated double integral and evaluate the integral.

1. The coordinate axes and the line $x + y = 2$.

4. The parabola $x = y - y^2$ and the line $y = -x$.

14. $\displaystyle\int_0^3 \int_{-x}^{x(2-x)} dydx.$

18. $\displaystyle\int_0^2 \int_{x^2-4}^{0} dydx + \int_0^4 \int_0^{\sqrt{x}} dydx.$

21. Find the average height of the paraboloid $z = x^2 + y^2$ over the square $0 \le x \le 2, 0 \le y \le 2$.

22. Find the average value of $f(x, y) = 1/(xy)$ over the square $\ln 2 \le x \le 2\ln 2, \ln 2 \le y \le 2\ln 2$

THOMAS' CALCULUS (12/E)

15.4 Double Integrals in Polar Form

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1 Integrals in Polar Coordinates

1.1 Integrals are sometimes easier to evaluate if we change to _____.

1.2 Suppose that a function _____ is defined over a region R that is bounded by the rays _____ and _____, and by the continuous curves _____ and _____, where $0 \leq g_1(\theta) \leq g_2(\theta) \leq a$, and $\alpha \leq \theta \leq \beta$.

圖示如下:

1.3 (a) The polar rectangles that lie inside R, calling their areas _____. Let _____ be any point in the polar rectangle whose area is _____.

(b) If f is continuous throughout R, the sum _____ will approach a limit as we refine the grid to make _____. The limit is called the double integral of f over R:

$$\lim_{n\to\infty} S_n = \underline{\qquad\qquad\qquad}$$

1.4 Write the sum S_n that expresses ΔA_k in terms of Δr and $\Delta\theta$.

圖示如下:

(a) Let the kth polar rectangle be _____. Let _____ be the average of the radii of the _____ and _____ bounding the kth polar rectangle ΔA_k.

(b) The area of a wedge-shaped sector of a circle having radius r and angle θ: _____.

(c) The area of the circular sectors:
Inner radius: _____, Outer radius: _____

(d) A_k =area of large sector − area of small sector

$=$ _____ $=$ _____

(e) $S_n =$ _____. $\lim_{n\to\infty} S_n =$ _____.

1.5 The double integral of f over R is _____

$$\iint_R f(r,\theta)\ dA = \underline{\qquad\qquad\qquad\qquad}$$

1.6 The area of a closed and bounded region R in the polar coordinate plane is

$$A =$$

1.7 *Changing Cartesian Integrals into Polar Integrals*

$$\iint_R f(x,y)\ dx\ dy = \underline{\qquad\qquad\qquad\qquad}\ ,$$

where G denotes the same region of integration.

Ex. 1 ..... (`example2, p855`)

Find the area enclosed by the lemniscate $r^2 = 4\cos 2\theta$.

sol:

Ex. 2 ..... (example3, p856)

Evaluate $\iint\limits_R e^{x^2+y^2}\ dy\ dx$ where R is the semicircular region bounded by the x-axis and the curve $y = \sqrt{1-x^2}$.

sol:

Ex. 3 ..... (example4, p856)

Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2+y^2)\ dy\ dx$.

sol:

Ex. 4 ..... (example5, p856)

Find the volume of the solid region bounded above bby the paraboloid $z = 9-x^2-y^2$ and below by the unit circle in the xy-plane.

sol:

實習課練習 (EXERCISE 15.5)

9. $\displaystyle\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} dy\ dx.$

17. $\displaystyle\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{0} \frac{2}{1+\sqrt{x^2+y^2}}\ dy\ dx$

19. $\displaystyle\int_{0}^{\ln 2}\int_{0}^{\sqrt{(\ln 2)^2-y^2}} e^{\sqrt{x^2+y^2}}\ dx\ dy$

20. $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{-\sqrt{1-y^2}} \ln(x^2+y^2+1)\ dx\ dy$

27. Find the area of the region cut from the first quadrant by the curve $r = 2(2 - \sin 2\theta)^{1/2}$

28. Find the area of the region that lies inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$.

105-2 小考 (1) 題目

2017/03/20, Calculus Quiz (1), §7.1 ∼ §7.5 (交回題目卷及答案卷)(每題 30 分)

1. (a) Show that if f has an interval I as domain and $f'(x)$ exists and is never zero on I, then $(f^{-1})'(b) = \frac{1}{f'(f^{-1}(b))}$ where $a = f^{-1}(b)$. (b) Let $f(x) = x^3 - 3x^2 - 1, x \geq 2$. Find the value of df^{-1}/dx at $x = -1 = f(3)$ without finding a formula for $f^{-1}(x)$.

2. (a) Find dy/dx. (a) $y = \displaystyle\int_{x^2/2}^{x^2} \log_5 \sqrt{t}\, dt$. (b) $y = \sqrt[3]{\dfrac{x^2(x-1)}{x^2+1}}$.

3. (a) Show that the number e can be calculated as the limit. (b) Evaluate $\displaystyle\int_2^4 x^{2x}(1+\ln x)\, dx$.

4. Find the limit. (a) $\displaystyle\lim_{x\to 1^+}\left(\frac{1}{x-1} - \frac{1}{\ln x}\right)$. (b) $\displaystyle\lim_{x\to e^+}(\ln x)^{1/(x-e)}$.

105-2 小考 (2) 題目

2017/04/10, Calculus Quiz (2), §7.6 ∼ §8.3(交回題目卷及答案卷) (每小題 15 分，共 105 分)

1. $\displaystyle\int \frac{1}{(\sin^{-1} x)\sqrt{1-x^2}}\, dx.$

2. (a) $\displaystyle\int \sqrt{\frac{x}{1-x^3}}\, dx.$ (b) $\displaystyle\int \frac{x^3}{\sqrt{x^2+4}}\, dx.$

3. (a) Show that $\displaystyle\int x^n \sin x\, dx = -x^n \cos x + n \int x^{n-1} \cos x\, dx.$ (b) $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx.$

4. (a) $\displaystyle\int \sin^2 2x \cos^3 2x\, dx$ (b) $\displaystyle\int \sec^4 x \tan^2 x\, dx.$

105-2 小考 (3) 題目

2017/05/15, Calculus Quiz (3), §10.1 ∼ §10.8(交回題目卷及答案卷) (共 100 分)

1. (15 分) Find $\lim_{n\to\infty} a_n$ if the sequence $\{a_n = (\frac{x^n}{2n+1})^{1/n}, x > 0\}$ converges.

2. (30 分) Determine the convergence or divergence: (a)$\sum_{n=1}^{\infty} \frac{n^2}{e^{n/3}}$ (b) $\sum_{n=1}^{\infty} \left(\ln\left(e^2 + \frac{1}{n}\right)\right)^{n+1}$.

3. (30 分) Determine the convergence or divergence: (a) $\sum_{n=1}^{\infty} \frac{n2^n(n+1)!}{3^n n!}$ (b) $\sum_{n=1}^{\infty} \frac{1}{1+2+3+\cdots+n}$

4. (25 分) Find the series's radius and interval of convergence. For what values of x does the series converge absolutely, or conditionally? $\sum_{n=0}^{\infty} \frac{(-1)^n(x+2)^n}{n}$.

105-2 小考 (4) 題目

2017/06/12, Calculus Quiz (4), §14.2 ∼ §14.4, §15.1 ∼ §15.3 (交回題目卷及答案卷) (共 100 分)

1. (20%) (a) $\displaystyle\lim_{\substack{(x,y)\to(2,2)\\ x+y\neq 4}} \frac{x+y-4}{\sqrt{x+y}-2}$. (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\cos(x^2+y^2)}{x^2+y^2}$.

2. (20%) Find $\partial f/\partial x$ and $\partial f/\partial y$ if $f(x,y)=\displaystyle\sum_{n=1}^{\infty}(e^{xy})^n$ and $|e^{xy}|<1$.

3. (30%) (a) Evaluate dw/dt at $t=1$: $w=2ye^x-\ln z$, $x=\ln(t^2+1)$, $y=\tan^{-1}t$, $z=e^t$.

   (b) Find $\partial z/\partial x$ and $\partial z/\partial y$ at (0,0,0) if $x^3+z^2+ye^{xz}+z\sin xy=0$.

4. (30%) (a) $\displaystyle\int_0^2\int_x^2 2y^2\sin xy\ dy\ dx$. (b) $\displaystyle\int_0^{\ln 2}\int_0^{\sqrt{(\ln 2)^2-y^2}} e^{\sqrt{x^2+y^2}}\ dx\ dy$.

國立臺北大學 105 學年度第 2 學期 期中 考試命題紙

考試科目： 微積分　　開課班別： 通訊 1/電機 1/智財學程　　命題教授: 吳漢銘

考試日期：4 月 17 日 (一) 10:10~11:40	※准帶項目打「O」，否則打「×」					1. 需加發計算紙或答案紙請在試題內封袋備註。 2. 為環保節能減碳，試題一律採雙面印刷，如有特殊印製需求，請註記：
本試題共 1 頁，印刷份數: 80 份	計算機	課本	筆記	電子辭典	紙本字典	
備註：注意事項要看!!	×	×	×	×	×	

注意事項: (1) 答案卷請寫上姓名及學號。(2) 請按題號順序書寫。(3) 每一題號需置於答案卷最左邊。(4) 可用鉛筆。(5) 需要計算過程。(6) 同時交回答案卷、題目卷、計算紙。(7) 總分共 120 分。

1. (10 分) Find dy/dx $y = \int_{x^2/2}^{x^2} \log_5 \sqrt{t}\, dt$.

2. (10 分) $\lim_{x \to e^+} (\ln x)^{1/(x-e)}$.

3. (10 分) $\int \sqrt{\frac{x}{1-x^3}}\, dx$.

4. (10 分) $\int \frac{x^3}{\sqrt{x^2+4}}\, dx$.

5. (10 分) Derive the integration by parts formula.

6. (10 分) What is the definition of the improper integrals (type I and type II)?

7. (10 分) Using trigonometric substitutions to evaluate the integrals: $\int \frac{8\, dx}{(4x^2+1)^2}$.

8. (10 分) Use partial fractions to evaluate $\int \frac{2x+2}{(x^2+1)(x-1)^3}\, dx$.

9. (10 分) Evaluating improper integrals: $\int_0^2 \frac{dx}{\sqrt{|x-1|}}$.

10. (10 分) Testing for convergence: $\int_1^\infty \frac{1}{e^x - 2^x}\, dx$.

11. (20 分) Find the values of p for which the integral $\int_2^\infty \frac{dx}{x(\ln x)^p}$ converges.

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國立臺北大學 105 學年度第 2 學期 期末 考試命題紙

考試科目： 微積分　　開課班別： 通訊 1/電機 1/智財學程　　命題教授: 吳漢銘

考試日期：6 月 19 日 (一) 10:10~11:40	※准帶項目打「O」，否則打「×」					1. 需加發計算紙或答案紙請在試題內封袋備註。 2. 為環保節能減碳，試題一律採雙面印刷，如有特殊印製需求，請註記：
本試題共 1 頁，印刷份數: 80 份	計算機	課本	筆記	電子辭典	紙本字典	
備註：注意事項要看!!	×	×	×	×	×	

注意事項: (1) 答案卷請寫上姓名及學號。(2) 請按題號順序書寫。(3) 每一題號需置於答案卷最左邊。(4) 可用鉛筆。(5) 需要計算過程。(6) 同時交回答案卷、題目卷、計算紙。(7) 總分共 110 分。

1. (10 分) Find the series's radius and interval of convergence. For what values of x does the series converge absolutely, or conditionally? $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 3^{2n}(x-2)^n}{3n}$.

2. (10 分) $\displaystyle\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\, dydx$.

3. (20 分) (a) Plot the following points given in a polar coordinates. Then find all the polar coordinates of each point. $(3, \pi/4)$, $(-3, \pi/4)$, $(3, -\pi/4)$, $(-3, -\pi/4)$. (b) Replace the Cartesian equation $(x+2)^2 + (y-5)^2 = 16$ by equivalent polar equations.

4. (20 分) Let $\vec{v} = -\vec{i} + \vec{j}$, $\vec{u} = \sqrt{2}\,\vec{i} + \sqrt{3}\,\vec{j} + 2\vec{k}$, find Find (a) $\vec{v}\cdot\vec{u}$, $\|\vec{v}\|$, $\|\vec{u}\|$; (b) the cosine of the angle between $\vec{v}$ and $\vec{u}$. (c) the scalar component of $\vec{u}$ in the direction of $\vec{v}$. (d) the vector $\text{proj}_{\vec{v}}\vec{u}$.

5. (10 分) Let $f(x,y) = \begin{cases} \sqrt{y^3}, & y \geq 0 \\ -y^2, & y < 0. \end{cases}$ Using the definition of the partial derivative of $f(x,y)$ to find $f_y(x,0)$.

6. (20 分) (a) Suppose that $F(x,y)$ is differentiable and that the equation $F(x,y) = 0$ defines y as a differentiable function of x. Show that, at any point where $F_y \neq 0$, $dy/dx = -F_x/F_y$.
(b) Suppose that the equation $F(x,y,z) = xe^y + ye^z + 2\ln x - 2 - 3\ln 2 = 0$ defines the variable z implicitly as a function $z = f(x,y)$. Find the values of $\partial z/\partial x$ and $\partial z/\partial y$ at the point $(1, \ln 2, \ln 3)$.

7. (10 分) Find the volume of the solid in the first octant (八分圓) bounded by the coordinate planes, the plane $x = 3$, and the parabolic (拋物線的) cylinder (圓柱) $z = 4 - y^2$.

8. (加分題 10 分) (a) 本課程助教姓名: _____。(b) 助教的研究室在哪一間或助教都在哪一間教室回答同學們的問題: _____。

注意： 1、考試求公平及公正，請同學務必自律，維護學校與學生之榮譽。
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2018/03/26, Calculus Quiz (1), §7.2 ∼ §7.6 (交回題目卷及答案卷)(每題 30 分)

1. (a) Find dy/dx. (a) $y = \int_{x^2/2}^{x^2} \log_5 \sqrt{t}\ dt$. (b) $y = \sqrt[3]{\dfrac{x^2(x-1)}{x^2+1}}$.

2. (a) Show that the number e can be calculated as the limit. (b) Evaluate $\int_2^4 x^{2x}(1+\ln x)\ dx$.

3. Find the limit. (a) $\lim_{x\to 1^+}(\dfrac{1}{x-1} - \dfrac{1}{\ln x})$. (b) $\lim_{x\to e^+}(\ln x)^{1/(x-e)}$.

4. Evaluate the integrals. (a) $\int \dfrac{1}{(\sin^{-1} x)\sqrt{1-x^2}}\ dx$. (b) $\int \dfrac{dx}{(x+1)\sqrt{x^2+2x}}$.

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2018/03/26, Calculus Quiz (1), §7.2 ∼ §7.6 (交回題目卷及答案卷)(每題 30 分)

1. (a) Find dy/dx. (a) $y = \int_{x^2/2}^{x^2} \log_5 \sqrt{t}\ dt$. (b) $y = \sqrt[3]{\dfrac{x^2(x-1)}{x^2+1}}$.

2. (a) Show that the number e can be calculated as the limit. (b) Evaluate $\int_2^4 x^{2x}(1+\ln x)\ dx$.

3. Find the limit. (a) $\lim_{x\to 1^+}(\dfrac{1}{x-1} - \dfrac{1}{\ln x})$. (b) $\lim_{x\to e^+}(\ln x)^{1/(x-e)}$.

4. Evaluate the integrals. (a) $\int \dfrac{1}{(\sin^{-1} x)\sqrt{1-x^2}}\ dx$. (b) $\int \dfrac{dx}{(x+1)\sqrt{x^2+2x}}$.

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2018/03/26, Calculus Quiz (1), §7.2 ∼ §7.6 (交回題目卷及答案卷)(每題 30 分)

1. (a) Find dy/dx. (a) $y = \int_{x^2/2}^{x^2} \log_5 \sqrt{t}\ dt$. (b) $y = \sqrt[3]{\dfrac{x^2(x-1)}{x^2+1}}$.

2. (a) Show that the number e can be calculated as the limit. (b) Evaluate $\int_2^4 x^{2x}(1+\ln x)\ dx$.

3. Find the limit. (a) $\lim_{x\to 1^+}(\dfrac{1}{x-1} - \dfrac{1}{\ln x})$. (b) $\lim_{x\to e^+}(\ln x)^{1/(x-e)}$.

4. Evaluate the integrals. (a) $\int \dfrac{1}{(\sin^{-1} x)\sqrt{1-x^2}}\ dx$. (b) $\int \dfrac{dx}{(x+1)\sqrt{x^2+2x}}$.

2018/06/04, Calculus Quiz (3), §8.7 ∼ §10.5 (交回題目卷及答案卷)(每題 30 分)

1. Determine the convergence of the series $\sum_{n=2}^{\infty}(-1)^n \frac{\ln n}{n - \ln n}$. (converges absolutely, converge conditionally or diverges)

2. Find the series' radius and interval of convergence. For what values of x does the series converge absolutely, or conditionally?
   (a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x+2)^n}{n2^n}$, (b) $\sum_{n=2}^{\infty} \frac{x^n}{n \ln n}$.

3. Find the Taylor series generated by $f = 2^x$ at $x = 1$.

4. Let $f(x,y) = \begin{cases} \sqrt{x}, & x \geq 0 \\ x^2, & x < 0. \end{cases}$ Find f_x, f_y, f_{xy} and f_{yx} and state the domain for each partial derivative.

5. Assuming that the equation $xe^y + ye^z + 2\ln x - 2 - 3\ln 2 = 0$ defined y as a differentiable function of x, find the value of dy/dx at the point $(1, \ln 2, \ln 3)$.

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2018/06/04, Calculus Quiz (3), §8.7 ∼ §10.5 (交回題目卷及答案卷)(每題 30 分)

1. Determine the convergence of the series $\sum_{n=2}^{\infty}(-1)^n \frac{\ln n}{n - \ln n}$. (converges absolutely, converge conditionally or diverges)

2. Find the series' radius and interval of convergence. For what values of x does the series converge absolutely, or conditionally?
   (a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x+2)^n}{n2^n}$, (b) $\sum_{n=2}^{\infty} \frac{x^n}{n \ln n}$.

3. Find the Taylor series generated by $f = 2^x$ at $x = 1$.

4. Let $f(x,y) = \begin{cases} \sqrt{x}, & x \geq 0 \\ x^2, & x < 0. \end{cases}$ Find f_x, f_y, f_{xy} and f_{yx} and state the domain for each partial derivative.

5. Assuming that the equation $xe^y + ye^z + 2\ln x - 2 - 3\ln 2 = 0$ defined y as a differentiable function of x, find the value of dy/dx at the point $(1, \ln 2, \ln 3)$.

國立臺北大學 106 學年度第 2 學期 期中 考試命題紙

考試科目： 微積分　　開課班別： 通訊 1/電機 1/智財學程　　命題教授: 吳漢銘

考試日期：4 月 16 日 (一) 10:10~11:40	※准帶項目打「O」，否則打「×」					1. 需加發計算紙或答案紙請在試題內封袋備註。 2. 為環保節能減碳，試題一律採雙面印刷，如有特殊印製需求，請註記：
本試題共 1 頁，印刷份數: 77 份	計算機	課本	筆記	電子辭典	紙本字典	
備註：注意事項要看!!	×	×	×	×	×	

注意事項: (1) 答案卷請寫上姓名及學號。(2) 請按題號順序書寫。(3) 每一題號需置於答案卷最左邊。(4) 可用鉛筆。(5) 需要計算過程。(6) 同時交回答案卷、題目卷、計算紙。(7) 每題 10 分，總分共 100 分。

1. (a) Show that the number e can be calculated as the limit. (b) Evaluate $\int_2^4 x^{2x}(1+\ln x)\,dx$.

2. Find the limit. (a) $\lim_{x\to 1^+}(\frac{1}{x-1}-\frac{1}{\ln x})$. (b) $\lim_{x\to e^+}(\ln x)^{1/(x-e)}$.

3. $\int \cos\sqrt{x}\,dx$

4. $\int \sqrt{x}e^{\sqrt{x}}\,dx$

5. $\int \sin^3 x\cos^3 x\,dx$

6. $\int_{-\pi}^{\pi}(1-\cos^2 t)^{3/2}\,dt$

7. $\int \frac{x^2}{(x^2-1)^{5/2}}\,dx$

8. $\int \frac{x}{\sqrt{1+x^4}}\,dx$

9. $\int \frac{9x^3-3x+1}{x^3-x^2}\,dx$

10. $\int \frac{1}{x^6(x^5+4)}\,dx$ (hint: let $u=x^5$)

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國立臺北大學 106 學年度第 2 學期 期末 考試命題紙

考試科目： 微積分　　開課班別： 通訊 1/電機 1/智財學程　　命題教授: 吳漢銘

考試日期：6 月 25 日 (一) 10:10~11:40	※准帶項目打「O」，否則打「×」					1. 需加發計算紙或答案紙請在試題內封袋備註。 2. 為環保節能減碳，試題一律採雙面印刷，如有特殊印製需求，請註記：
本試題共 1 頁，印刷份數: 77 份	計算機	課本	筆記	電子辭典	紙本字典	
備註：注意事項要看!!	×	×	×	×	×	

注意事項: (1) 答案卷請寫上姓名及學號。(2) 請按題號順序書寫。(3) 每一題號需置於答案卷最左邊。(4) 可用鉛筆。(5) 需要計算過程。(6) 同時交回答案卷、題目卷、計算紙。(7) 總分共 110 分。

1. (10 分) Determine the convergence of the series (converge absolutely, converge conditionally, or diverge). $\sum_{n=1}^{\infty} \frac{(-1)^n(n^2+1)}{2n^2+n-1}$

2. (10 分) (a) Find the series' radius and interval of convergence. Then identify the values of x for which the series converges (b) absolutely and (c) conditionally. $\sum_{n=0}^{\infty} \frac{(-1)^n(x-1)^{2n+1}}{2n+1}$.

3. (10 分) Find the Taylor series generated by $f(x) = 2^x$ at $a = 1$.

4. (10 分) Assuming that the equation $2xy + e^{x+y} - 2 = 0$ define y as a differentiable function of x, find the value of dy/dx at point $P(0, \ln 2)$.

5. (10 分) Assume that $w = f(s^3 + t^2)$ and $f'(x) = e^x$. Find $\frac{\partial w}{\partial t}$ and $\frac{\partial w}{\partial s}$.

6. (15 分) Write an iterated integral for $\iint_R dA$ over the described region $R = \{$bounded by $y = x^2$ and $y = x + 2\}$ using (a) vertical cross-sections, (b) horizontal cross-sections.

7. (15 分) Evaluate the integral: $\int_0^2 \int^{4-x^2} \frac{xe^{2y}}{4-y}\, dydx$.

8. (15 分) Find the volume of the solid in the first octant (八分圓) bounded by the coordinate planes, the plane $x = 3$, and the parabolic (拋物線的) cylinder (圓柱) $z = 4 - y^2$.

9. (15 分) Find the average value of $f(x, y) = xy$ over the regions: the quarter circle $x^2 + y^2 \leq 1$ in the first quadrant.

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2019/03/18, Calculus Quiz (1), §7.5 ∼ §8.1 (可用鉛筆、需計算過程、交回題目卷及答案卷)

1. (30%) (a) $\lim_{x\to 1^+}\left(\frac{1}{x-1}-\frac{1}{\ln x}\right)$, (b) $\lim_{x\to 0}\frac{(e^x-1)^2}{x\sin x}$, (c) $\lim_{x\to\infty}(\frac{x^2+1}{x+2})^{1/x}$.

2. (10%) 導出分部積分 (Integration by parts) 的公式。

3. (30%) (a) $\int \frac{dx}{x\sqrt{25x^2-2}}$, (b) $\int_0^{\ln\sqrt{3}} \frac{e^x\ dx}{1+e^{2x}}$, (c) $\int \frac{dy}{(\sin^{-1}y)\sqrt{1-y^2}}$

4. (30%) (a) $\int e^\theta \sin\theta\ d\theta$, (b) $\int \ln(x+x^2)\ dx$, (c) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\ dx$

. . . . . .

2019/03/18, Calculus Quiz (1), §7.5 ∼ §8.1 (可用鉛筆、需計算過程、交回題目卷及答案卷)

1. (30%) (a) $\lim_{x\to 1^+}\left(\frac{1}{x-1}-\frac{1}{\ln x}\right)$, (b) $\lim_{x\to 0}\frac{(e^x-1)^2}{x\sin x}$, (c) $\lim_{x\to\infty}(\frac{x^2+1}{x+2})^{1/x}$.

2. (10%) 導出分部積分 (Integration by parts) 的公式。

3. (30%) (a) $\int \frac{dx}{x\sqrt{25x^2-2}}$, (b) $\int_0^{\ln\sqrt{3}} \frac{e^x\ dx}{1+e^{2x}}$, (c) $\int \frac{dy}{(\sin^{-1}y)\sqrt{1-y^2}}$

4. (30%) (a) $\int e^\theta \sin\theta\ d\theta$, (b) $\int \ln(x+x^2)\ dx$, (c) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\ dx$

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2019/03/18, Calculus Quiz (1), §7.5 ∼ §8.1 (可用鉛筆、需計算過程、交回題目卷及答案卷)

1. (30%) (a) $\lim_{x\to 1^+}\left(\frac{1}{x-1}-\frac{1}{\ln x}\right)$, (b) $\lim_{x\to 0}\frac{(e^x-1)^2}{x\sin x}$, (c) $\lim_{x\to\infty}(\frac{x^2+1}{x+2})^{1/x}$.

2. (10%) 導出分部積分 (Integration by parts) 的公式。

3. (30%) (a) $\int \frac{dx}{x\sqrt{25x^2-2}}$, (b) $\int_0^{\ln\sqrt{3}} \frac{e^x\ dx}{1+e^{2x}}$, (c) $\int \frac{dy}{(\sin^{-1}y)\sqrt{1-y^2}}$

4. (30%) (a) $\int e^\theta \sin\theta\ d\theta$, (b) $\int \ln(x+x^2)\ dx$, (c) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\ dx$

2019/05/06, Calculus Quiz (2), §10.4 ∼ §10.8 (可用鉛筆、需計算過程、交回題目卷及答案卷)

1. (20%) Use the Comparison Test or the Limit Comparison Test to determine if each series converges or diverges. (a) $\sum_{n=1}^{\infty}(\frac{2n+3}{5n+4})^n$. (b) $\sum_{n=1}^{\infty}\frac{1}{n3^n}$.

2. (20%) Determine the convergence of the series (conditionally, absolutely, or diverges)
   (a) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1+n}{n^2}$. (b) $\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}+\sqrt{n+1}}$.

3. (30%) Find the series's radius and interval of convergence. For what values of x does the series converge absolutely, or conditionally? (a) $\sum_{n=1}^{\infty}(1+\frac{1}{n})^n x^n$. (b) $\sum_{n=0}^{\infty}(\sqrt{n+1}-\sqrt{n})(x-3)^n$.

4. (10%) Find the Maclaurin series of xe^x.

5. (20%) Find the Taylor series and the Taylor polynomials generated by $f(x)=\sin x$ at $x=0$.

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2019/05/06, Calculus Quiz (2), §10.4 ∼ §10.8 (可用鉛筆、需計算過程、交回題目卷及答案卷)

1. (20%) Use the Comparison Test or the Limit Comparison Test to determine if each series converges or diverges. (a) $\sum_{n=1}^{\infty}(\frac{2n+3}{5n+4})^n$. (b) $\sum_{n=1}^{\infty}\frac{1}{n3^n}$.

2. (20%) Determine the convergence of the series (conditionally, absolutely, or diverges)
   (a) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1+n}{n^2}$. (b) $\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}+\sqrt{n+1}}$.

3. (30%) Find the series's radius and interval of convergence. For what values of x does the series converge absolutely, or conditionally? (a) $\sum_{n=1}^{\infty}(1+\frac{1}{n})^n x^n$. (b) $\sum_{n=0}^{\infty}(\sqrt{n+1}-\sqrt{n})(x-3)^n$.

4. (10%) Find the Maclaurin series of xe^x.

5. (20%) Find the Taylor series and the Taylor polynomials generated by $f(x)=\sin x$ at $x=0$.

2019/05/27, Calculus Quiz (3), §14.4 ∼ §15.2 (可用鉛筆、需計算過程、交回題目卷及答案卷)

1. (10%) $\displaystyle\lim_{\substack{(x,y)\to(2,2)\\ x+y\neq 4}} \frac{x+y-4}{\sqrt{x+y}-2}$.

2. (20%) Let $f(x,y) = \begin{cases} y\sqrt{x}, & x \geq 0, y \in R \\ yx^2, & x < 0, y \in R. \end{cases}$ Find f_x, f_y, f_{xy} and f_{yx} and state the domain for each partial derivative.

3. (30%) (a) Find dy/dx at the given point: $xe^y + \sin xy + y - \ln 2 = 0$, $(0, \ln 2)$. (b) Evaluate $\partial z/\partial u$ and $\partial z/\partial v$ at the given point (u,v): $z = \tan^{-1}(x/y)$, $x = u\cos v$, $y = u\sin v$, $(u,v) = (1.3, \pi/6)$.

4. (20%) Find the volume of the wedgelike solid that lies beneath the surface $z = 16 - x^2 - y^2$ and above the region R bounded by the curve $y = 2\sqrt{x}$, the line $y = 4x - 2$, and the x-axis.

5. (20%) (a) $\displaystyle\int_0^{\pi/6}\int_{\sin x}^{1/2} xy^2\, dydx$. (b) $\displaystyle\int_0^1\int_y^1 x^2 e^{xy}\, dxdy$.

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2019/05/27, Calculus Quiz (3), §14.4 ∼ §15.2 (可用鉛筆、需計算過程、交回題目卷及答案卷)

1. (10%) $\displaystyle\lim_{\substack{(x,y)\to(2,2)\\ x+y\neq 4}} \frac{x+y-4}{\sqrt{x+y}-2}$.

2. (20%) Let $f(x,y) = \begin{cases} y\sqrt{x}, & x \geq 0, y \in R \\ yx^2, & x < 0, y \in R. \end{cases}$ Find f_x, f_y, f_{xy} and f_{yx} and state the domain for each partial derivative.

3. (30%) (a) Find dy/dx at the given point: $xe^y + \sin xy + y - \ln 2 = 0$, $(0, \ln 2)$. (b) Evaluate $\partial z/\partial u$ and $\partial z/\partial v$ at the given point (u,v): $z = \tan^{-1}(x/y)$, $x = u\cos v$, $y = u\sin v$, $(u,v) = (1.3, \pi/6)$.

4. (20%) Find the volume of the wedgelike solid that lies beneath the surface $z = 16 - x^2 - y^2$ and above the region R bounded by the curve $y = 2\sqrt{x}$, the line $y = 4x - 2$, and the x-axis.

5. (20%) (a) $\displaystyle\int_0^{\pi/6}\int_{\sin x}^{1/2} xy^2\, dydx$. (b) $\displaystyle\int_0^1\int_y^1 x^2 e^{xy}\, dxdy$.

國立臺北大學 107 學年度第 2 學期 期中 考試命題紙

考試科目： 微積分　　開課班別： 資工 1　　命題教授: 吳漢銘

考試日期：4 月 15 日 (一) 10:10~11:40	※准帶項目打「O」，否則打「×」					1. 需加發計算紙或答案紙請在試題內封袋備註。 2. 為環保節能減碳，試題一律採雙面印刷，如有特殊印製需求，請註記：
本試題共 1 頁，印刷份數: 105 份	計算機	課本	筆記	電子辭典	紙本字典	
備註：注意事項要看!! (§7.5~§10.3)	×	×	×	×	×	

注意事項: (1) 答案卷請寫上姓名及學號。(2) 請按題號順序書寫。(3) 每一題號 (含小題) 需置於答案卷最左邊。(4) 可用鉛筆。(5) 需要計算過程。(6) 同時交回答案卷、題目卷、計算紙。(7) 總分共 114 分。

1. (16 分) (a) $\lim_{x\infty}(\frac{x^2+1}{x+2})^{1/x}$.　　(b) $\int \ln(x+x^2)\ dx$,

2. (16 分) (a) $\int_0^{\pi/2} \sin^2 2\theta \cos^3 2\theta\ d\theta$.　　(b) $\int_0^{\pi/2} \theta\sqrt{1-\cos 2\theta}\ d\theta$.

3. (16 分) (a) $\int \frac{v^2}{(1-v^2)^{5/2}}\ dv$.　　(b) $\int \frac{x\ dx}{\sqrt{1+x^4}}$.

4. (16 分) (a) $\int \frac{8x^2+8x+2}{(4x^2+1)^2}\ dx$.　　(b) $\int \frac{e^t\ dt}{e^{2t}+3e^t+2}$.

5. (16 分) (a) $\int_{-\infty}^{\infty} \frac{2x\ dx}{(x^2+1)^2}$.　　(b) $\int_2^4 \frac{dt}{t\sqrt{t^2-4}}$.

6. (8 分) Does the integral $\int_1^{\infty} \frac{1}{\sqrt{e^x-x}}$ converge?

7. (10 分) Discuss (with proofs) the convergence of the p-series.

8. (16 分) Determining convergence or divergence. (a) $\sum_{n=1}^{\infty} \frac{n^2}{e^{n/3}}$.　　(b) $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{\ln n}$.

注意： 1、考試求公平及公正，請同學務必自律，維護學校與學生之榮譽。

2、考試時不得有交談、窺視、夾帶、抄襲、傳遞、代考或其它作弊等舞弊行為，考畢務必交卷，不得攜卷出場，違者依考場規則議處。

國立臺北大學 107 學年度第 2 學期 期末 考試命題紙

考試科目： 微積分　　開課班別： 資工 1　　命題教授: 吳漢銘

考試日期：06 月 17 日 (一) 10:10~11:40	※准帶項目打「O」，否則打「×」					1. 需加發計算紙或答案紙請在試題內封袋備註。 2. 為環保節能減碳，試題一律採雙面印刷，如有特殊印製需求，請註記：
本試題共 1 頁，印刷份數: 100 份	計算機	課本	筆記	電子辭典	紙本字典	
備註：注意事項要看!!	×	×	×	×	×	A 卷

注意事項: (1) 答案卷請寫上姓名及學號。(2) 請按題號順序書寫。(3) 每一題號需置於答案卷最左邊。(4) 可用鉛筆。(5) 需要計算過程。(6) 同時交回答案卷、題目卷、計算紙。(7) 總分共 100 分。(8) 請於答案卷的記分欄下方，填上卷別「A」或「B」。

1. (10 分) Given the series $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{n}+\sqrt{n+1}}$, determine the convergence of the series (conditionally, absolutely, or diverges).

2. (10 分) Given the series $\sum_{n=0}^{\infty}(\sqrt{n+1}-\sqrt{n})(x-3)^n$, find the series's radius and interval of convergence. For what values of x does the series converge absolutely, or conditionally?

3. (10 分) Find the volume of the wedgelike solid that lies beneath the surface $z = 16 - x^2 - y^2$ and above the region R bounded by the curve $y = 2\sqrt{x}$, the line $y = 4x - 2$, and the x-axis.

4. (10 分) $\int_0^{\pi/6} \int_{\sin x}^{1/2} xy^2 \, dydx$.

5. (10 分) Replace the following Cartesian equation by equivalent polar equations.
(a) $xy = 2$. (b) $x^2 + xy + y^2 = 1$.

6. (20 分) Graph the curve $r^2 = 4\cos 2\theta$.

7. (20 分) Find the area of the region inside the circle $r = 3a\cos\theta$ and outside the cardioid $r = a(1+\cos\theta)$, $a > 0$.

8. (10 分) Express the region's area as an iterated double integral and evaluate the integral: The parabola $x = y - y^2$ and the line $y = -x$.

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2020/03/30, 108-2 微積分小考 (1), §7.3 ∼ §8.1 (可用鉛筆、需計算過程、交回題目卷及答案卷)

1. (30%) (a) Show that $e = \lim_{x\to 0}(1+x)^{1/x}$. (b) $\displaystyle\int \frac{1}{1+e^x}\,dx$.
2. (20%) Apply l'Hôpital's Rule to show that $\lim_{x\to 0^+}(1+x)^{1/x} = e$.
3. (10%) Find the limit: $\displaystyle\lim_{x\to\infty}\frac{e^{x^2}}{xe^x}$.
4. (10%) Find the derivative of $y = \sin^{-1} x$.
5. (30%) (a) Find $\displaystyle\int e^x \sin x\,dx$. (b) Find $\displaystyle\int x^5 e^x\,dx$.

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2020/03/30, 108-2 微積分小考 (1), §7.3 ∼ §8.1 (可用鉛筆、需計算過程、交回題目卷及答案卷)

1. (30%) (a) Show that $e = \lim_{x\to 0}(1+x)^{1/x}$. (b) $\displaystyle\int \frac{1}{1+e^x}\,dx$.
2. (20%) Apply l'Hôpital's Rule to show that $\lim_{x\to 0^+}(1+x)^{1/x} = e$.
3. (10%) Find the limit: $\displaystyle\lim_{x\to\infty}\frac{e^{x^2}}{xe^x}$.
4. (10%) Find the derivative of $y = \sin^{-1} x$.
5. (30%) (a) Find $\displaystyle\int e^x \sin x\,dx$. (b) Find $\displaystyle\int x^5 e^x\,dx$.

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2020/03/30, 108-2 微積分小考 (1), §7.3 ∼ §8.1 (可用鉛筆、需計算過程、交回題目卷及答案卷)

1. (30%) (a) Show that $e = \lim_{x\to 0}(1+x)^{1/x}$. (b) $\displaystyle\int \frac{1}{1+e^x}\,dx$.
2. (20%) Apply l'Hôpital's Rule to show that $\lim_{x\to 0^+}(1+x)^{1/x} = e$.
3. (10%) Find the limit: $\displaystyle\lim_{x\to\infty}\frac{e^{x^2}}{xe^x}$.
4. (10%) Find the derivative of $y = \sin^{-1} x$.
5. (30%) (a) Find $\displaystyle\int e^x \sin x\,dx$. (b) Find $\displaystyle\int x^5 e^x\,dx$.

2020/05/25, 108-2 微積分小考 (2), §10.3 ∼ §10.8 (可用鉛筆、需計算過程、交回題目卷及答案卷)

1. Use the Ratio Test or Root Test to determine if each series converges or diverges.
   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{10^n}$. (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n \ln n}{2^n}$.

2. Determine the convergence of the series, absolutely, conditionally or diverge?
   (a) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n - \ln n}$. (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{(2n)!}{2^n n! n}$.

3. Find the series's radius and interval of convergence. Foe what values of x does the series converge absolutely, or conditionally? (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (x+2)^n}{n}$. (b) $\displaystyle\sum_{n=1}^{\infty} \frac{3 \cdot 5 \cdot 7 \cdots (2n+1)}{n^2 \cdot 2^n} x^{n+1}$.

4. (a) Find the Maclaurin series: xe^x.
   (b) Find the Taylor series and the Taylor polynomials generated by $f(x) = \sin x$ at $x = 0$.

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2020/05/25, 108-2 微積分小考 (2), §10.3 ∼ §10.8 (可用鉛筆、需計算過程、交回題目卷及答案卷)

1. Use the Ratio Test or Root Test to determine if each series converges or diverges.
   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{10^n}$. (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n \ln n}{2^n}$.

2. Determine the convergence of the series, absolutely, conditionally or diverge?
   (a) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n - \ln n}$. (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{(2n)!}{2^n n! n}$.

3. Find the series's radius and interval of convergence. Foe what values of x does the series converge absolutely, or conditionally? (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (x+2)^n}{n}$. (b) $\displaystyle\sum_{n=1}^{\infty} \frac{3 \cdot 5 \cdot 7 \cdots (2n+1)}{n^2 \cdot 2^n} x^{n+1}$.

4. (a) Find the Maclaurin series: xe^x.
   (b) Find the Taylor series and the Taylor polynomials generated by $f(x) = \sin x$ at $x = 0$.

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2020/05/25, 108-2 微積分小考 (2), §10.3 ∼ §10.8 (可用鉛筆、需計算過程、交回題目卷及答案卷)

1. Use the Ratio Test or Root Test to determine if each series converges or diverges.
   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{10^n}$. (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n \ln n}{2^n}$.

2. Determine the convergence of the series, absolutely, conditionally or diverge?
   (a) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n - \ln n}$. (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{(2n)!}{2^n n! n}$.

3. Find the series's radius and interval of convergence. Foe what values of x does the series converge absolutely, or conditionally? (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (x+2)^n}{n}$. (b) $\displaystyle\sum_{n=1}^{\infty} \frac{3 \cdot 5 \cdot 7 \cdots (2n+1)}{n^2 \cdot 2^n} x^{n+1}$.

4. (a) Find the Maclaurin series: xe^x.
   (b) Find the Taylor series and the Taylor polynomials generated by $f(x) = \sin x$ at $x = 0$.

國立臺北大學 108 學年度第 2 學期 期中 考試命題紙

考試科目： 微積分　　開課班別： 通訊 1/電機 1/電資院 1　　命題教授: 吳漢銘

考試日期：04 月 27 日 (一) 10:10~11:40	※准帶項目打「O」，否則打「×」					1. 需加發計算紙或答案紙請在試題內封袋備註。 2. 為環保節能減碳，試題一律採雙面印刷，如有特殊印製需求，請註記：
本試題共 1 頁，印刷份數: 65 份	計算機	課本	筆記	電子辭典	紙本字典	
備註：注意事項要看!! (§7.3~§10.2)	×	×	×	×	×	

注意事項: (1) 答案卷請寫上姓名及學號。(2) 請按題號順序書寫。(3) 每一題號需置於答案卷最左邊。(4) 可用鉛筆。(5) 需要計算過程。(6) 同時交回答案卷、題目卷、計算紙。(7) 每題 10 分，共 100 分。

1. Find the limit: $\lim_{x\to\infty} \dfrac{e^{x^2}}{xe^x}$.

2. $\displaystyle\int \frac{1}{1+e^x}\,dx$.

3. Evaluate $\displaystyle\int \sin^2 x \cos^4 x\,dx$.

4. Evaluate $\displaystyle\int \frac{dx}{\sqrt{25x^2-4}}$, $\quad x > 2/5$.

5. Express the integrands as a sum of partial fractions and evaluate the integrals. $\displaystyle\int \frac{8x^2+8x+2}{(4x^2+1)^2}\,dx$.

6. For what values of p does the integral $\displaystyle\int_1^\infty \frac{1}{x^p}\,dx$ converge? When the integral does converge, what is its value?

7. Evaluate the integral. $\displaystyle\int_0^2 \frac{dx}{\sqrt{|x-1|}}$.

8. Test the integrals for convergence. $\displaystyle\int_{-1}^1 \ln|x|\,dx$.

9. Which of the sequence $\{a_n\}$ converge, and which diverge? Find the limit of each convergent sequence. (a) $a_n = (\dfrac{x^n}{2n+1})^{1/n}$, $\quad x > 0$. (b) $a_n = \sqrt[n]{n^2+n}$.

10. Establish the reduction formula: $\displaystyle\int x^n \sin x\,dx = -x^n\cos x + n\int x^{n-1}\cos x\,dx$.

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國立臺北大學 108 學年度第 2 學期 期末 考試命題紙

考試科目： 微積分　　開課班別： 通訊 1/電機 1/電資院 1　　命題教授: 吳漢銘

考試日期：06 月 22 日 (一) 10:10~11:40	※准帶項目打「O」，否則打「×」					1. 需加發計算紙或答案紙請在試題內封袋備註。 2. 為環保節能減碳，試題一律採雙面印刷，如有特殊印製需求，請註記：
本試題共 1 頁，印刷份數: 65 份	計算機	課本	筆記	電子辭典	紙本字典	
備註：注意事項要看!! (§10.3~§15.3)	×	×	×	×	×	

注意事項: (1) 答案卷請寫上姓名及學號。(2) 請按題號順序書寫。(3) 每一題號需置於答案卷最左邊。(4) 可用鉛筆。(5) 需要計算過程。(6) 同時交回答案卷、題目卷、計算紙。(7) 每題 10 分，共 100 分。

1. Plot the following points given in a polar coordinates. Then find all the polar coordinates of each point. (a) $P(-2, \pi/2)$, (b) $Q(3, -\pi/4)$.

2. (a) Replace the polar equation, $r^2 \sin 2\theta = 2$, by the equivalent Cartesian equation. (b) Replace the Cartesian equation, $x^2 + xy + y^2 = 1$, by the equivalent polar equation.

3. Find the slope of the curve $r = -1 + \cos\theta$; $\theta = \pi/2$.

4. Graph the curve $r^2 = 4\cos 2\theta$.

5. Let $f(x, y) = \begin{cases} \sqrt{x}, & x \geq 0 \\ x^2, & x < 0. \end{cases}$ Find f_x, f_y, f_{xy} and f_{yx} and state the domain for each partial derivative.

6. Find dy/dx at the given point: $xe^y + \sin xy + y - \ln 2 = 0$, $(0, \ln 2)$.

7. $\displaystyle\int_0^{\pi/6} \int_{\sin x}^{1/2} xy^2 \, dydx.$

8. Find the volume of the solid in the first octant bounded by the coordinate planes, the plane $x = 3$, and the parabolic cylinder $z = 4 - y^2$.

9. $\displaystyle\int_{-1}^{0} \int_{-2x}^{1-x} dydx + \int_0^2 \int_{-x/2}^{1-x} dydx.$

10. Find the average value of $f(x, y) = 1/(xy)$ over the square $\ln 2 \leq x \leq 2\ln 2, \ln 2 \leq y \leq 2\ln 2$.

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微積分會考-考古題

科　　目：微積分

考試時間：100 分鐘

※ 注意：

（一）本試題共有單選題 20 題(選項：五選一)，填空題 10 題。

（二）選擇題請選出一個正確或最適當的答案，用 2B 鉛筆依題號於試卡上清楚劃記，複選作答者，該題不予計分。

（三）填空題請將正確答案依題號填於答案卷空格中。

（四）本試題不可使用電子計算器。

（五）本試題不得攜帶出考場。

一、選擇題：

1. Calculate $\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}=$

   (A) ∞ (B) $-\infty$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) $\frac{1}{\sqrt{2}}$

2. Choose an equation from the following that expresses the fact that a function f is continuous at the number 6.

   (A) $\lim_{x\to 0} f(x)=6$ (B) $\lim_{x\to 0} f(x)=f(6)$ (C) $\lim_{x\to 6} f(x)=6$

   (D) $\lim_{x\to 6} f(x)=f(0)$ (E) $\lim_{x\to 6} f(x)=f(6)$

3. Let $f(x)=\begin{cases} x+7 & , x<3 \\ -x^2+9x-7, & x\geq 3 \end{cases}$. Which one of the following statements is TRUE ?

   (A) $\lim_{x\to 3^-} f(x)=11$ (B) $\lim_{x\to 3^+} f(x)=11$ (C) $\lim_{x\to 3} f(x)=11$

   (D) $f(3)=10$ (E) f is continuous at 3

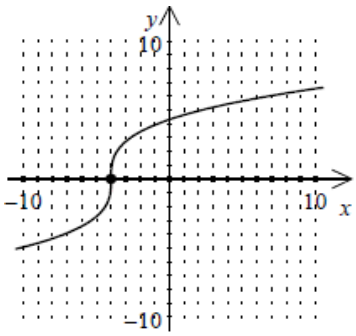
4. Determine where the function is differentiable.

   (A) $x=-4$ (B) $y=0$

   (C) $y\neq 5$

   (D) All x except $x=-4$

   (E) The function is differentiable everywhere.



微積分會考-考古題

5. If $\lim_{x\to c}\frac{f(x)-f(c)}{x-c}=a$, $a\in$ ¡ , which one of the following statements is FALSE?
   (A) $f(c)=0$ (B) $f'(c)=a$ (C) f is differentiable at c
   (B) f is continuous at c (E) $\lim_{x\to c} f(x)=f(c)$

6. Find the equation of the tangent line to the curve $f(x)=x^3-x^2+6$ at the point (1, 6).
   (A) $y=2x+4$ (B) $y=x+5$ (C) $2y=x+11$ (D) $y=-x+7$
   (E) $y=-2x+8$

7. Evaluate $\frac{d}{dx}\int_0^1 e^{x^2}dx=$
   (A) e^{x^2} (B) $2xe^{x^2}$ (C) $e-1$ (D) $2e$ (E) 0

8. The annual sales x of a new consumer product can be modeled by $x=\frac{15,000t^2}{36+t^2}$, where t is the number of years since the introduction of the product. Find the time at which the sales are increasing most rapidly.
   (A) $2\sqrt{3}$ years (B) $3\sqrt{2}$ years (C) $3\sqrt{3}$ years
   (D) $3\sqrt{5}$ years (E) $2\sqrt{6}$ years

9. Let $f(x)=x\ln x$, $x>0$. Then the minimum value of $f(x)$ is:
   (A) 0 (B) 1 (c) e (D) $-e$ (E) $-\frac{1}{e}$

10. Evaluate $\int_2^5 [x]dx$, where [x] is the largest integral less than or equal to x.
   (A) 7 (B) 8 (C) 9 (D) 10 (E) 10.5

11. The indefinite integral $\int x^2\ln 5x dx$
   (A) $\frac{x^2}{2}(2\ln 5x-1)+c$ (B) $\frac{x^3}{3}(3\ln 5x+1)+c$ (C) $\frac{x^2}{4}(2\ln 5x+1)+c$
   (D) $\frac{x^3}{9}(3\ln 5x-1)+c$ (E) $\frac{x^2}{15}(2\ln 5x-1)+c$

12. The indefinite integral of $\int\frac{1}{x^2-1}dx$ is :
   (A) $\frac{1}{2}\ln|\frac{x-1}{x+1}|+c$ (B) $\ln|x^2-1|+c$ (C) $\frac{1}{x}\ln|x^2-1|+c$
   (D) $\tan^{-1}x+c$ (E) $\sec^{-1}x+c$

微積分會考-考古題

13. Find the particular solution, $y = f(x)$, that satisfies the conditions

$f''(x) = \frac{6}{\sqrt{x}} + 3$, $f'(1) = 12$, $f(4) = 56$.

(A) $f(x) = 12x\sqrt{x} + \frac{3}{2}x^2 + 6x - 88$ (B) $f(x) = 3x\sqrt{x} + \frac{3}{2}x^2 + 7x - 20$

(C) $f(x) = 8x\sqrt{x} + \frac{3}{2}x^2 - 3x - 20$ (D) $f(x) = 12\sqrt[3]{x} + \frac{3}{2}x^2 + 6x - 8$

(E) $f(x) = 3\sqrt[3]{x} + \frac{3}{2}x^2 + 6x - 8$

14. The definite integral of $\int_{-1}^{1}(e^{-x}+1)dx$ is:

(A) $-e^{-x} + x + c$ (B) $e - e^{-1} + 2$ (C) $e + e^{-1}$ (D) 2 (E) 0

15. Find the area of the region bounded by the graphs $f(y) = y^2 + 1$ and $g(y) = 4 - 2y$.

(A) $\frac{31}{3}$ (B) $\frac{23}{4}$ (C) $\frac{31}{4}$ (D) $\frac{32}{3}$ (E) $\frac{22}{3}$

16. If f is continuous and $\int_0^4 f(x)dx = 10$, find $\int_0^2 f(2x)dx$.

(A) 5 (B) 10 (C) 15 (D) 20 (E) 25

17. The indefinite integral of $\int e^{-(x-\theta)}dx$ is

(A) $-\frac{1}{x-\theta}e^{-(x-\theta)} + c$ (B) $-\frac{1}{\theta}e^{-(x-\theta)} + c$ (C) $-\frac{1}{x}e^{-(x-\theta)} + c$

(D) $-\theta e^{-(x-\theta)} + c$ (E) $-e^{-(x-\theta)} + c$

18. Find the limit of $\lim_{x\to\infty}(1+\frac{2}{x})^{5x}$.

(A) 1 (B) e^5 (C) e^{10} (D) ∞ (E) 0

19. Let $f(x, y) = y^3 - 3yx^2 - 3y^2 - 3x^2 + 1$. Which one of the following points is NOT a critical point of f ?

(A) $(\sqrt{3}, -1)$ (B) $(-\sqrt{3}, 1)$ (C) $(-\sqrt{3}, -1)$ (D) $(0, 2)$ (E) $(0, 0)$

20. Evaluate the value of $\int_0^2\int_0^x xydydx$?

(A) $\frac{1}{2}$ (B) $\frac{4}{3}$ (C) 1 (D) 2 (D) 4

微積分會考-考古題

二、填空題：

1. Let $f(x)=\dfrac{x^2+x-6}{|x-2|}$, $x\neq 2$. Then $\lim\limits_{x\to 2^-} f(x)=$_____

2. $\lim\limits_{x\to\infty}(\sqrt{x^2+x+1}-\sqrt{x^2-x+1})=$_____

3. Suppose that $f'(a)=2$, then $\lim\limits_{h\to 0}\dfrac{f(a+2h)-(a-h)}{h}=$_____

4. $\lim\limits_{x\to 0^+} x\ln x=$_____

5. Evaluate $\displaystyle\int\frac{x}{\sqrt{x^2+1}}dx=$_____

6. The improper integral of $\displaystyle\int_1^\infty\frac{\ln x}{x^2}dx=$_____

7. The area of the region bounded by the curves $y=e^x$ and $y=e^{-x}$ between $x=0$ and $x=\ln 5$ is: _____

8. If $F(X)=\displaystyle\int_0^{\ln x}\sqrt{e^t+1}dt$, then $F'(x)=$_____

9. Let $f(x,y)=y\ln x+2xy^2$. The directional derivative of f at $(1,-2)$ is: _____

10. Evaluate the improper integral $\displaystyle\int_e^\infty\frac{1}{x(\ln x)^3}dx$_____.

微積分會考-考古題

一、選擇題答案

1	2	3	4	5
C	E	B	D	A
6	7	8	9	10
B	E	A	E	C
11	12	13	14	15
D	A	C	B	D
16	17	18	19	20
A	E	C	B	D

二、填空題答案

1	2	3	4	5
-5	1	6	0	$\sqrt{x^2+1}+C$
6	7	8	9	10
1	$\frac{16}{5}$	$\frac{\sqrt{x+1}}{x}$	$[6,\ -8]$	$\frac{1}{2}$

108 年度微積分會考試題 (B)

科　　目：微積分

考試時間：100 分鐘

※ 注意：

（一）本試題共有單選題 20 題(選項：五選一)，填空題 10 題。

（二）選擇題請選出一個正確或最適當的答案，複選作答者，該題不予計分。

（三）填空題請將正確答案依題號填於答案卷空格中。

（四）本試題不可使用電子計算器。

（五）本試題不得攜帶出考場。

一、選擇題：

1. Let $f(x)=\begin{cases} x^2-1 & \text{if } x>1 \\ 2x-1 & \text{if } x\le 1 \end{cases}$. Which one of the following statements is **TRUE**?

(A) $\lim_{x\to 1^-} f(x)=0$　(B) $\lim_{x\to 1^-} f(x)=1$　(C) $\lim_{x\to 1^+} f(x)=1$

(D) $f(x)$ is continuous at 1.　(E) $f(x)$ is differentiable at 1.

2. Determine the range of the given function $f(x)=\begin{cases} x^2-1 & \text{if } x>1 \\ 3 & \text{if } x=1 \\ 2-5x & \text{if } x<1 \end{cases}$.

(A) $(-\infty, \infty)$　(B) $(-3, 3]$　(C) $[-3, 3]$　(D) $(-3, \infty)$　(E) $(-\infty, 3]$

3. Determine the domain of the given function $f(t)=\dfrac{\sqrt{t^2-1}}{t-3}$.

(A) $(-\infty, \infty)$　(B) $[-1, 1]$　(C) $(-\infty, -1]\cup[1, \infty)$

(D) $(-\infty, 3)\cup(3, \infty)$　(E) $(-\infty, -1]\cup[1, 3)\cup(3, \infty)$

4. Find the equation of the tangent line to the curve $f(x)=x^3-x^2+6$ at the point (1, 6).

(A) $y=2x+4$　(B) $y=-2x+8$　(C) $y=x+5$

(D) $y=5x+1$　(E) $y=-x+7$

5. Find $\lim_{x\to 4}\dfrac{\sqrt{x}-2}{x-4}$.

(A) $\dfrac{1}{4}$　(B) 0　(C) 2　(D) 6　(E) not exist

6. If $h(x)=(x^4-2)(x^3-1)(x^2+1)$, then $h'(1)$ equals :

(A) 3　(B) 6　(C) −6　(D) 10　(E) −4

108 年度微積分會考試題 (B)

7. Let $y=\ln(x^2+x+1)$. $\dfrac{dy}{dx}=$

(A) e^{x^2+x+1} (B) $(2x+1)e^{x^2+x+1}$ (C) $\dfrac{1}{x^2+x+1}$

(D) $\dfrac{2x+1}{x^2+x+1}$ (E) $y=\ln|2x+1|$

8. Which one of the following is a horizontal asymptote of $f(x)=\dfrac{x^2}{x^2-1}$?

(A) $y=1$ (B) $y=-1$ (C) $y=0$

(D) $x=1$ (E) $x=-1$

9. If $h(x)=\dfrac{x}{x-7}$, $x\neq 7$, then the graph of $h(x)$ is

(A) decreasing on $(-\infty,\ \infty)$

(B) decreasing on $(-\infty,\ 7)$ and $(7,\ \infty)$

(C) increasing on $(-\infty,\ 7)$ and $(7,\ \infty)$

(D) decreasing on $(-\infty,\ 7)$ and increasing on $(7,\ \infty)$

(E) increasing on $(-\infty,\ 7)$ and decreasing on $(7,\ \infty)$

10. Let $f(x)=2x^3-3x^2+1$. Which one of the following statements is **TRUE**?

(A) $f(x)$ has a relative maximum at $\dfrac{1}{2}$. (B) $f(x)$ has a relative minimum at $\dfrac{1}{2}$.

(C) $f(x)$ has a relative minimum at 0. (D) $f(x)$ has a relative minimum at 1.

(E) $(0,\ 1)$ is a point if inflection of $f(x)$

11. Which one of the following statements related to $f(x)=xe^x$ is **FALSE**

(A) $f'(x)=(x+1)e^x$ (B) $f''(x)=(x+2)e^x$ (C) $\lim\limits_{x\to-\infty} f(x)=0$

(D) $\min\{f(x)\,|\,x\in\mathbb{R}\}=-\dfrac{1}{e}$ (E) $(0,\ 0)$ is a point of inflection.

12. A farmer plans to build a 24 meters long fence beside the bank of a river. A rectangle area will be surrounded by the fence in 3 sides and the river to be the forth side. The maximum area of the rectangle is:

(A) 18 square meters (B) 24 square meters (C) 36 square meters

(D) 64 square meters (E) 72 square meters

13. Suppose that you deposit P dollars in an account whose annual interest rate is 5%, compounded continuously. How long will it take for your balance to double?

(A) 20 years (B) $20\sqrt{e}$ years (C) $20\ln 2$ years

(D) $5e^2$ years (E) $2\ln 5$ years

108 年度微積分會考試題 (B)

14. Evaluate the integral $\int_0^1 \frac{1}{1+e^{-x}}dx$.

(A) $\ln\frac{1+e}{2}$ (B) $\ln(e+1)$ (C) $e(e-1)$

(D) $\frac{1}{1+e}$ (E) $e(e-1)$

15. If $F(x)=\int_0^{x^2} e^{-\sqrt{t}}dt$, $x>0$, then $F'(x)$ is :

(A) $x^2e^{-\sqrt{x}}$ (B) $2xe^{-x}$ (C) x^2e^{-x} (D) $2\sqrt{x}e^{-\sqrt{x}}$ (E) $2xe^{-\sqrt{x}}$

16. Find the volume of the solid of revolution obtained by revolving the region under the curve, $y=\sqrt{x+2}$, from $x=0$ to $x=5$ about the x-axis.

(A) $\frac{9\pi}{2}$ (B) $\frac{13\pi}{3}$ (C) $\frac{81\pi}{2}$ (D) $\frac{45\pi}{2}$ (E) $\frac{81\pi}{3}$

17. Evaluate the integral $\int_2^5 |x-3|\,dx$.

(A) 0.5 (B) 1.5 (C) 2.5 (D) 3.5 (E) 4.5

18. The integral $\int_0^\infty \frac{x}{x^2+1}dx$ is :

(A) 0 (B) $\frac{1}{2}\ln 2$ (C) $\ln 2$ (D) $-\ln 2$ (E) ∞

19. If f is continuous and $\int_0^4 f(x)dx=10$, then $\int_0^2 f(2x)dx$ is:

(A) 5 (B) 10 (C) 15 (D) 20 (E) 25

20. The indefinite integral $\int x\ln x dx$ is:

(A) $x\ln x+c$ (B) $x\ln x-x+c$ (C) $x\ln x+x+c$

(D) xe^x+c (E) e^x+x+c

108 年度微積分會考試題 (B)

二、填空題：

1. Let $f(x)=\dfrac{e^x-e^{-x}}{2}$. $f''(x)=$_____.

.

2. Let $f(x)=x^5+2x^3+3x+1$ be a one-to-one function. $(f^{-1})'(7)=$_____.

3. $\lim\limits_{x\to\infty}\dfrac{e^x-e^{-x}}{e^x+e^{-x}}=$_____.

4. The point inflection of $f(x)=x^3+x+1$ is _____.

5. The definite integral $\int_0^\infty x^2e^{-x}dx=$_____.

6. If $f(x)=\ln(x+1)$, then $\lim\limits_{x\to1}\dfrac{f(x)-f(1)}{x-1}=$_____.

7. The area of the region bounded by the graphs of the functions $y=x^2$ and $y=\sqrt[3]{x}$ is _____ .

8. The indefinite integral $\int\dfrac{x^2-3}{x}dx=$_____.

9. $\dfrac{d}{dx}x^x=$_____.

10. $\lim\limits_{n\to\infty}\left(1-\dfrac{3}{n}\right)^n=$_____