2020 / 10 / 12 微積分小考 (1) . §2.1 ~ §2.5

＊符號標錯 (-2) ; 說明不清楚 (-3)

1. (a) $P(2,4)$, $Q(2+h, (2+h)^2)$

$$\frac{\Delta y}{\Delta x} = \frac{(2+h)^2-2^2}{(2+h)-2} = \frac{h^2+4h}{h} = h+4 \ ,\ h \to 0,\ \lim_{h\to 0}(h+4)=4 \ \#$$ (10分)

$$y-4 = 4(x-2) \Rightarrow y = 4(x-2)+4 = 4x-4 \ \#$$ (5分)

沒寫或亂寫計算過程 (-5)

1. (b) $$\frac{\Delta g}{\Delta x} = \frac{g(1+h)-g(1)}{(1+h)-1} = \frac{\sqrt{1+h}-1}{h} \ \#$$ (7分)

$$\lim_{h\to 0}\frac{\sqrt{1+h}-1}{h} = \lim_{h\to 0}\frac{\sqrt{1+h}-1}{h}\left(\frac{\sqrt{1+h}+1}{\sqrt{1+h}+1}\right) = \lim_{h\to 0}\frac{(1+h)-1}{h(\sqrt{1+h}+1)} = \lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}(\sqrt{1+h}+1)} = \frac{1}{2} \ \#$$ (8分)

2. ① $y=\begin{cases} 0 , & x<0 \\ 1 , & x\ge 0\end{cases}$ ② $y=\begin{cases} \frac{1}{x} , & x\ne 0 \\ 0 , & x=0\end{cases}$ ③ $y=\begin{cases} 0 , & x\le 0 \\ \sin\frac{1}{x} , & x>0\end{cases}$ (5分/個)

④ $\underline{y=\tan x}$ (3分) , $\underline{當\ x\to \frac{\pi}{2}, \frac{3}{2}\pi, \frac{5}{2}\pi \ldots\ 時}$ (2分)

[$f(x)$: 函數 ⇐ 寫這個
$\lim f(x)$: 函數的極限]

3. (a)

Let $f(x)$ be defined on an open interval about x_0, except possibly at x_0 itself. We say that the limit of $f(x)$ as x approaches x_0 is the number L, and we write $\underline{\lim_{x\to x_0} f(x) = L}$. (3分)
if, for every number $\underline{\varepsilon>0}$ (1分), there exists a corresponding number $\underline{\delta>0}$ (1分)
such that $\underline{\text{for all } x}$ (1分), $\underline{0<|x-x_0|<\delta}$ (2分) $\Rightarrow$ $\underline{|f(x)-L|<\varepsilon}$ (2分)

3. (b).

* 答案為 $\delta = \frac{\varepsilon}{1+\varepsilon}$!!

5分 (I) $\left|\frac{1}{x}-1\right| < \varepsilon \Rightarrow -\varepsilon < \frac{1}{x}-1 < \varepsilon \Rightarrow 1-\varepsilon < \frac{1}{x}-1 < 1+\varepsilon \Rightarrow \frac{1}{1+\varepsilon} < x < \frac{1}{1-\varepsilon}$

5分 (II) $|x-1| < \delta \Rightarrow -\delta < x-1 < \delta \Rightarrow 1-\delta < x-1 < 1+\delta$

5分 Then, $1-\delta = \frac{1}{1+\varepsilon} \Rightarrow \delta = 1-\frac{1}{1+\varepsilon} = \frac{\varepsilon}{1+\varepsilon}$, or $1+\delta = \frac{1}{1-\varepsilon} \Rightarrow \delta = \frac{1}{1-\varepsilon}-1 = \frac{\varepsilon}{1-\varepsilon}$

Choose $\delta = \frac{\varepsilon}{1+\varepsilon}$, the smaller of the two distances.

5分 $\forall \varepsilon, \exists \delta = \frac{\varepsilon}{1+\varepsilon}$, s.t. $\lim_{x\to 1} \frac{1}{x} = 1$

4. (a)

$$\lim_{x\to 1^+} \frac{\sqrt{2x}(x-1)}{|x-1|} = \lim_{x\to 1^+} \frac{\sqrt{2x}(x-1)}{(x-1)} = \lim_{x\to 1^+} \sqrt{2x} = \sqrt{2} \quad (5分)$$

$$\lim_{x\to 1^-} \frac{\sqrt{2x}(x-1)}{|x-1|} = \lim_{x\to 1^-} \frac{\sqrt{2x}(x-1)}{-(x-1)} = \lim_{x\to 1^-} -\sqrt{2x} = -\sqrt{2} \quad (5分)$$

$\lim_{x\to 1^+} \frac{\sqrt{2x}(x-1)}{|x-1|} \neq \lim_{x\to 1^-} \frac{\sqrt{2x}(x-1)}{|x-1|}$，故 $\lim_{x\to 1} \frac{\sqrt{2x}(x-1)}{|x-1|}$ 不存在 # (5分).

4. (b) (10分)

$$\lim_{x\to 0} \frac{x+x\cos x}{\sin x\cos x} = \lim_{x\to 0}\left(\frac{x}{\sin x\cos x} + \frac{x\cos x}{\sin x\cos x}\right)$$

$$= \lim_{x\to 0}\left(\frac{x}{\sin x}\cdot\frac{1}{\cos x}\right) + \lim_{x\to 0}\frac{x}{\sin x}$$

$$= \lim_{x\to 0}\left(\frac{1}{\frac{\sin x}{x}}\right)\cdot \lim_{x\to 0}\left(\frac{1}{\cos x}\right) + \lim_{x\to 0}\left(\frac{1}{\frac{\sin x}{x}}\right)$$

$$= 1\cdot 1 + 1 = 2 \quad \#$$