

2020.10.26 : 微積分小考 (2) : 52.5 ~ 53.2.

* 符號標錯 (-2) * 過程不清楚, 亂寫 (扣該部分一半分數).

1.(a) If f is continuous function on a closed interval $[a, b]$ (1分)

and if y_0 is any value between $f(a)$ and $f(b)$, (1分)

then $y_0 = f(c)$ for some c in $[a, b]$ (1分/1分).

** $[a, b]$: $a \leq c \leq b$; (a, b) : $a < c < b$.

1.(b) $x^2 + \sqrt{2x+5} - 4 = 0 \Rightarrow x^2 + \sqrt{2x+5} = 4$

5分 $f(x) = x^2 + \sqrt{2x+5}$ is continuous on $[-\frac{5}{2}, \infty)$

$f(0) = \sqrt{5}$, $f(2) = 7$, f is also continuous on $[0, 2] \subset [-\frac{5}{2}, \infty)$,

5分 $y_0 = 4$, $\sqrt{5} \leq y_0 \leq 7$.

5分 By I.V.T., $f(c) = 4$, for some c in $[0, 2]$.

2.(a) The derivative of the function $f(x)$ with respect to the variable x is the function f' whose value at x is provided the limit exists.

$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ (5分), 沒寫 $f'(x)$ 扣 2 分

2.(b) $f'(x)$ 續 (a)

If f' exist at a particular x , we say that f is differentiable (has a derivative) at x .

3.(a) $f(x) = \frac{1-x^2}{x^2+1} = \frac{2}{x^2+1} - 1$, $\lim_{x \rightarrow \infty} f(x) = -1$, $\lim_{x \rightarrow -\infty} f(x) = -1$, $y = -1$ #

(b) $f(x) = \frac{x^2-4}{x-1} = x+1 - \frac{3}{x-1}$, $y = x+1$ # $x=1$ #

上述答案有錯 [(a) 扣 20 分, (b) 各扣 10 分], 多寫其他的 (扣 5 分).

4. ① $\frac{d}{dx}(1 \times 1) = \frac{d}{dx} x = 1$; ② $\frac{d}{dx}(1 \times 1) = \frac{d}{dx} (-x) = -1$

③ $\lim_{h \rightarrow 0^+} \frac{10+h-10}{h} = \lim_{h \rightarrow 0^+} \frac{1h}{h} = \lim_{h \rightarrow 0^+} \frac{h}{h} = 1$

④ $\lim_{h \rightarrow 0^-} \frac{10+h-10}{h} = \lim_{h \rightarrow 0^-} \frac{1h}{h} = \lim_{h \rightarrow 0^-} \frac{-h}{h} = -1$

(各 5 分) * 可微分和導數不存在皆要說明, 少一個扣 10 分.