2019/12/02, Calculus Quiz (3), §3.9 ～ §4.5

滿分為 100 分，整體批改標準：說明不清楚都是扣 3 分，符號標錯扣 2 分，其他批改標準於各小題解答後#處。

1.(10%)

$$f(x)=x^{1/3}\ \Rightarrow\ f'(x)=\frac{1}{3x^{2/3}}$$

$$\Rightarrow L(x)=f'(-8)(x-(-8))+f(-8)=\tfrac{1}{12}(x+8)-2$$

$$\Rightarrow L(x)=\tfrac{1}{12}x-\tfrac{4}{3}$$

2.(10%)

$$y=\frac{2\sqrt{x}}{3\left(1+\sqrt{x}\right)}=\frac{2x^{1/2}}{3\left(1+x^{1/2}\right)}$$

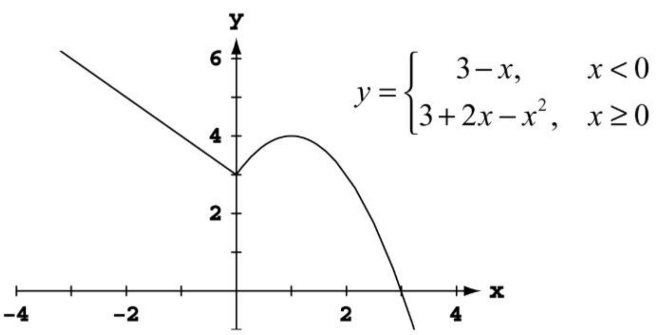
$$\Rightarrow dy=\left(\frac{x^{-1/2}\left(3\left(1+x^{1/2}\right)\right)-2x^{1/2}\left(\frac{3}{2}x^{-1/2}\right)}{9\left(1+x^{1/2}\right)^2}\right)dx=\frac{3x^{-1/2}+3-3}{9\left(1+x^{1/2}\right)^2}\,dx$$

$$\Rightarrow dy=\frac{1}{3\sqrt{x}\left(1+\sqrt{x}\right)^2}\,dx$$

3.(20%)

$$y'=\begin{cases}-1, & x<0\\ 2-2x, & x>0\end{cases}$$

crit. pt.	derivative	extremum	value
$x=0$	undefined	local min	3
$x=1$	0	local max	4



$[-4,4]$ by $[-1,6]$

local max/min 各佔 10 分

4.

(a)(15%)

Theorem 4: The Mean Value Theorem

Suppose that $y = f(x)$ is __continuous__ on a closed interval $[a, b]$ and __differentiable__ on the interval's interior (a, b). Then there is at least one point c in (a, b) at which __$\frac{f(b) - f(a)}{b - a} = f'(c)$__ .

(b)(15%)

Since f(x) must be continuous at x = 0 and x = 1 we have $\lim_{x \to 0^+} f(x) = a = f(0) \Rightarrow a = 3$ and $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) \Rightarrow -1 + 3 + a = m + b \Rightarrow 5 = m + b$. Since f(x) must also be differentiable at x = 1 we have $\lim_{x \to 1^-} f'(x) = \lim_{x \to 1^+} f'(x) \Rightarrow -2x + 3|_{x=1} = m|_{x=1} \Rightarrow 1 = m$. Therefore, a = 3, m = 1 and b = 4.

沒寫計算過程扣 10 分
計算過程不清楚扣 5 分

5.(30%)

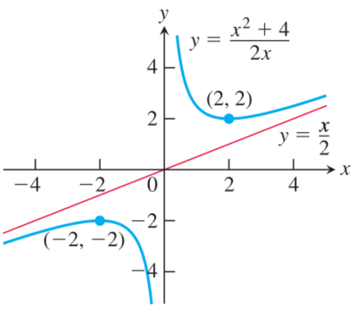


FIGURE 4.32 The graph of $y = \frac{x^2 + 4}{2x}$ (Example 9).

Solution

1. The domain of f is all nonzero real numbers. There are no intercepts because neither x nor $f(x)$ can be zero. Since $f(-x) = -f(x)$, we note that f is an odd function, so the graph of f is symmetric about the origin.
2. We calculate the derivatives of the function, but first rewrite it in order to simplify our computations:

$$f(x) = \frac{x^2 + 4}{2x} = \frac{x}{2} + \frac{2}{x}$$ Function simplified for differentiation

$$f'(x) = \frac{1}{2} - \frac{2}{x^2} = \frac{x^2 - 4}{2x^2}$$ Combine fractions to solve easily $f'(x) = 0$.

$$f''(x) = \frac{4}{x^3}$$ Exists throughout the entire domain of f

3. The critical points occur at $x = \pm 2$ where $f'(x) = 0$. Since $f''(-2) < 0$ and $f''(2) > 0$, we see from the Second Derivative Test that a relative maximum occurs at $x = -2$ with $f(-2) = -2$, and a relative minimum occurs at $x = 2$ with $f(2) = 2$.

4. On the interval $(-\infty, -2)$ the derivative f' is positive because $x^2 - 4 > 0$ so the graph is increasing; on the interval $(-2, 0)$ the derivative is negative and the graph is decreasing. Similarly, the graph is decreasing on the interval $(0, 2)$ and increasing on $(2, \infty)$.

5. There are no points of inflection because $f''(x) < 0$ whenever $x < 0$, $f''(x) > 0$ whenever $x > 0$, and f'' exists everywhere and is never zero throughout the domain of f. The graph is concave down on the interval $(-\infty, 0)$ and concave up on the interval $(0, \infty)$.

6. From the rewritten formula for $f(x)$, we see that

$$\lim_{x \to 0^+} \left(\frac{x}{2} + \frac{2}{x} \right) = +\infty \quad \text{and} \quad \lim_{x \to 0^-} \left(\frac{x}{2} + \frac{2}{x} \right) = -\infty,$$

so the y-axis is a vertical asymptote. Also, as $x \to \infty$ or as $x \to -\infty$, the graph of $f(x)$ approaches the line $y = x/2$. Thus $y = x/2$ is an oblique asymptote.

7. The graph of f is sketched in Figure 4.32. ■

\# $f'(x), f''(x)$ 佔 10 分

\# 畫圖 10 分

\# 漸近線 10 分