

2019/10/21, Calculus Quiz (2), §2.6 ~ §3.2

滿分為 100 分，整體批改標準：說明不清楚都是扣3分，符號標錯扣2分，其他批改標準於各小題解答後#處。

1.

(10%)(a)

$$-\frac{1}{3\theta} \leq \frac{\cos \theta}{3\theta} \leq \frac{1}{3\theta} \Rightarrow \lim_{\theta \rightarrow -\infty} \frac{\cos \theta}{3\theta} = 0 \text{ by the Sandwich Theorem}$$

(10%)(b)

$$\lim_{r \rightarrow \infty} \frac{r + \cos \frac{1}{r}}{2r + 7 - 5 \sin r} = \lim_{r \rightarrow \infty} \frac{1 + (\cos \frac{1}{r}) \frac{1}{r}}{2 + \frac{7}{r} - 5(\frac{\sin r}{r})} = \lim_{r \rightarrow \infty} \frac{1 + 0}{2 + 0 - 0} = \frac{1}{2}$$

1. 只寫答案沒有過程扣 5 分

2.

(20%)(a)

Since the highest power of x in the numerator is 1 more than the highest power of x in the denominator, there is an oblique asymptote. $y = \frac{2x^{3/2} + 2x - 3}{\sqrt{x+1}} = 2x - \frac{3}{\sqrt{x+1}}$, thus the oblique asymptote is $y = 2x$.

(20%)(b)

$$y = \sqrt{\frac{x^2+9}{9x^2+1}}: \lim_{x \rightarrow \infty} \sqrt{\frac{x^2+9}{9x^2+1}} = \lim_{x \rightarrow \infty} \sqrt{\frac{1+\frac{9}{x^2}}{9+\frac{1}{x^2}}} = \sqrt{\frac{1+0}{9+0}} = \frac{1}{3} \text{ and } \lim_{x \rightarrow -\infty} \sqrt{\frac{x^2+9}{9x^2+1}} = \lim_{x \rightarrow -\infty} \sqrt{\frac{1+\frac{9}{x^2}}{9+\frac{1}{x^2}}} = \sqrt{\frac{1+0}{9+0}} = \frac{1}{3},$$

thus $y = \frac{1}{3}$ is a horizontal asymptote.

3.

(10%)(a)

Definition: Derivative Function

The derivative of the function $f(x)$ with respect to the variable x is the function f' whose value at x is

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists.

(10%)(b)

$f'(x)$ 續(a)

If f' exist at a particular x , we say that f is differentiable (has a derivative) at x .

4.(20%)

Slope at origin = $\lim_{h \rightarrow 0} \frac{f(0+h)-f(0)}{h} = \lim_{h \rightarrow 0} \frac{h^2 \sin(\frac{1}{h})}{h} = \lim_{h \rightarrow 0} h \sin(\frac{1}{h}) = 0 \Rightarrow$ yes, $f(x)$ does have a tangent at the origin with slope 0.

=>Yes, $f(x)$ does have a tangent at the origin with slope 0.

2.沒有用導數定義計算扣 10分