

2019/10/14, Calculus Quiz (1), §2.1 ~ §2.5

滿分為 100 分，整體批改標準：說明不清楚都是扣3分，符號標錯扣2分，其他批改標準於各小題解答後#處。

1.(10%)(a)

Instantaneous speed at t_0 = average speed during $[t_0, t_0 + h]$ as h approaches 0 .

或

The average rate of change of $y = f(x)$ with respect to x over the interval $[x_1, x_2]$ is $\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x_1 + h) - f(x_1)}{h}, \quad h \neq 0$

Instantaneous speed is $\lim_{h \rightarrow 0} \frac{f(x_1+h)-f(x_1)}{h}$.

1. $h \rightarrow 0$ 只能寫在 \lim 的下標，不可以當作文字說明，扣3分

2. 沒說明函數 $f(x)$ 或沒說明 $\frac{f(x_1+h)-f(x_1)}{h}$ 是什麼 扣3分

(10%)(b)

Theorem 4: The Sandwich Theorem (The Squeeze Theorem, The Pinching Theorem)

Suppose that $g(x) \leq f(x) \leq h(x)$ for all x in some open interval containing c , except possibly at $x = c$ itself. Suppose also that

$$\lim_{x \rightarrow c} g(x) = \lim_{x \rightarrow c} h(x) = L, \quad \text{then } \lim_{x \rightarrow c} f(x) = L .$$

(10%)(c)

Interior point: A function $y = f(x)$ is continuous at an interior point c of its domain if

$$\lim_{x \rightarrow c} f(x) = f(c) .$$

Endpoint: A function $y = f(x)$ is continuous at a left endpoint a or is continuous at a right endpoint b of its domain if

$$\lim_{x \rightarrow a^+} f(x) = f(a) \quad \text{or} \quad \lim_{x \rightarrow b^-} f(x) = f(b) .$$

或

Continuity Test

A function $f(x)$ is continuous at $x = c$ if and only if it meets the following three conditions.

(a) $f(c)$ exists : c lies in the domain of f .

(b) $\lim_{x \rightarrow c} f(x)$ exists : f has a limit as $x \rightarrow c$.

(c) $\lim_{x \rightarrow c} f(x) = f(c)$: the limit equals the function value.

(a) To define continuity at a point in a function's domain, we need to define continuity at an interior point (which involves a two-sided limit) and continuity at an endpoint (which involves a one-sided limit).

3. 滿足的條件要寫完整，缺一項扣3分

(10%)(d)

Definition: Limit of a Function

Let $f(x)$ be defined on an open interval about x_0 , except possibly at x_0 itself. We say that the limit of $f(x)$ as x approaches x_0 is the number L , and we write

$$\lim_{x \rightarrow x_0} f(x) = L$$

if, for every number $\epsilon > 0$, there exists a corresponding number $\delta > 0$ such that for all x ,

$$0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \epsilon$$

2.(20%)

$$(1) x < 1 \Rightarrow \delta = \frac{\epsilon}{2} \Rightarrow \lim_{x \rightarrow 1^-} f(x) = 2$$

$$(2) x \geq 1 \Rightarrow \delta = \frac{\epsilon}{6} \Rightarrow \lim_{x \rightarrow 1^+} f(x) = 2$$

$$\therefore \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) \therefore \lim_{x \rightarrow 1} f(x) = 2$$

4. 沒有使用 Precise definition 一律 **0分**

5. 要分段討論(1)和(2)，缺一部分**扣10分**

6. 沒寫出 $\lim_{x \rightarrow 1^-} f(x)$ 和 $\lim_{x \rightarrow 1^+} f(x)$ ，**扣3分**

3.

(10%)(a) 2

(10%)(b) $\frac{1}{2} - \sqrt{5}$

4.

(10%)(a) 2

(10%)(b) -3

7. 沒有計算過程或過程亂寫，**扣5分**

8. 極限值不一定等於函數值，不能將 x 趨近的值直接代入函數作運算，以

3.(a)為例：寫 $g(\sqrt{5}) = 2 \Rightarrow \lim_{x \rightarrow \sqrt{5}} g(x) = 2$ ，**扣3分**