2019/11/04, Calculus Midterm Exam, §2.1 ～ §3.7

滿分為 100 分，整體批改標準：說明不清楚都是扣 3 分，符號標錯扣 2 分，其他批改標準於各小題解答後#處。

1.

(10%)

Slope at origin $= \lim_{h \to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h \to 0} \frac{h^2 \sin\left(\frac{1}{h}\right)}{h} = \lim_{h \to 0} h \sin\left(\frac{1}{h}\right) = 0 \Rightarrow$ yes, f(x) does have a tangent at the origin with slope 0.

=> Yes, f(x) does have a tangent at the origin with slope 0.

沒有用導數定義計算扣 5 分

2.

(10%)

Since the highest power of x in the numerator is 1 more than the highest power of x in the denominator, there is an oblique asymptote. $y = \frac{2x^{3/2}+2x-3}{\sqrt{x}+1} = 2x - \frac{3}{\sqrt{x}+1}$, thus the oblique asymptote is y = 2x.

沒有計算過程扣 5 分

3.

(10%)

Step1: $\left|\frac{1}{x}-\frac{1}{2}\right| < \epsilon \Rightarrow -\epsilon < \frac{1}{x}-\frac{1}{2} < \epsilon \Rightarrow \frac{1}{2}-\epsilon < \frac{1}{x} < \frac{1}{2}+\epsilon \Rightarrow \frac{2}{1+2\epsilon} < x < \frac{2}{1-2\epsilon}$

Step2: $|x-2| < \delta \Rightarrow -\delta < x-2 < \delta \Rightarrow 2-\delta < x < 2+\delta$

Then $2-\delta = \frac{2}{1+2\epsilon} \Rightarrow \delta = 2-\frac{2}{1+2\epsilon} = \frac{4\epsilon}{1+2\epsilon}$,

or $2+\delta = \frac{2}{1-2\epsilon} \Rightarrow \delta = \frac{2}{1-2\epsilon} - 2 = \frac{4\epsilon}{1-2\epsilon}$

Choose $\delta = \frac{4\epsilon}{1+2\epsilon}$, the smaller of the two distance.

Step1: 4 分, Step2: 4 分, 結論: 2 分

沒有使用 Precise definition 一律 0 分

4.

(10%)

函數$f(x) = 2x^3 - 2x^2 - 2x + 1$為連續函數，一實數$u$滿足$f(a) < u < f(b)$，則存在$c \in$ 實數閉區間$[a, b]$使得$f(c) = u$，因要求解，此題取$u = 0$。

因為 Intermedsite Value Theorem 適用於連續函數，沒寫出函數為連續扣 2 分

5.

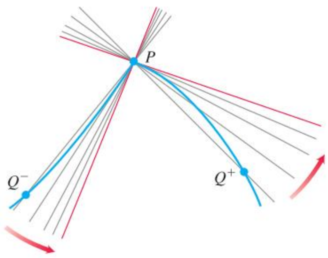
(10%)

(a) a corner, where the one-sided derivatives differ.

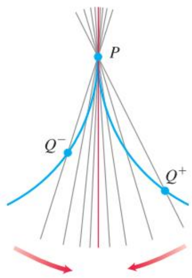
(b) a cusp, where the slope of PQ approaches ∞ from one side and $-\infty$ from the other.

(c) a vertical tangent, where the slope of PQ approaches ∞ from both sides or approaches $-\infty$ from both sides (here, $-\infty$).

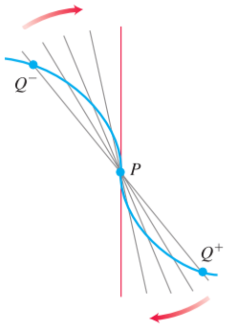
(d) a discontinuouity.



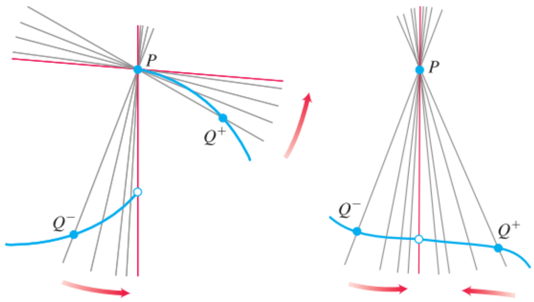
1. a *corner*, where the one-sided derivatives differ.



2. a *cusp*, where the slope of PQ approaches ∞ from one side and $-\infty$ from the other.



3. a *vertical tangent*, where the slope of PQ approaches ∞ from both sides or approaches $-\infty$ from both sides (here, $-\infty$).



4. a *discontinuity* (two examples shown).

寫對一個條件得 3 分

6.

(10%)

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \lim_{h\to 0}\frac{\dfrac{u(x+h)}{v(x+h)} - \dfrac{u(x)}{v(x)}}{h}$$

$$= \lim_{h\to 0}\frac{v(x)u(x+h) - u(x)v(x+h)}{hv(x+h)v(x)}$$

To change the last fraction into an equivalent one that contains the difference quotients for the derivatives of u and v, we subtract and add $v(x)u(x)$ in the numerator. We then get

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \lim_{h\to 0}\frac{v(x)u(x+h) - v(x)u(x) + v(x)u(x) - u(x)v(x+h)}{hv(x+h)v(x)}$$

$$= \lim_{h\to 0}\frac{v(x)\dfrac{u(x+h)-u(x)}{h} - u(x)\dfrac{v(x+h)-v(x)}{h}}{v(x+h)v(x)}.$$

只寫$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x)-f(x)g'(x)}{g^2(x)}$得 2 分

7.

(10%)

$$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x\dfrac{d}{dx}(\sin x) - \sin x\dfrac{d}{dx}(\cos x)}{\cos^2 x} \qquad \text{Quotient Rule}$$

$$= \frac{\cos x\cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x.$$

只寫$\frac{d}{dx}(tanx) = sec^2x$得 2 分

8.

(10%)

$$y' = \frac{3}{2}(1-\sqrt{sin3x})^{-2}(sin3x)^{-\frac{1}{2}}(cos3x)$$

$$y'' = 2(1-\sqrt{sin3x})^{-3}[\frac{3}{2}(sin3x)^{-\frac{1}{2}}(cos3x)]^2$$

$$-(1-\sqrt{sin3x})^{-2}(sin3x)^{-\frac{3}{2}}(\frac{3}{2}cos3x)^2 - \frac{9}{2}(1-\sqrt{sin3x})^{-2}\sqrt{sin3x}$$

y'和y''各 5 分

9.

(10%)

$g(x) = \frac{1}{x^2} - 1 \Rightarrow g'(x) = -\frac{2}{x^3} \Rightarrow g(-1) = 0$ and $g'(-1) = 2$; $f(u) = \left(\frac{u-1}{u+1}\right)^2 \Rightarrow f'(u) = 2\left(\frac{u-1}{u+1}\right)\frac{d}{du}\left(\frac{u-1}{u+1}\right)$
$= 2\left(\frac{u-1}{u+1}\right) \cdot \frac{(u+1)(1)-(u-1)(1)}{(u+1)^2} = \frac{2(u-1)(2)}{(u+1)^3} = \frac{4(u-1)}{(u+1)^3} \Rightarrow f'(g(-1)) = f'(0) = -4$; therefore,
$(f \circ g)'(-1) = f'(g(-1))g'(-1) = (-4)(2) = -8$

10.

(10%)

$$x^2 cos^2 3y - sin3y = 0$$
$$\Rightarrow x^2(2cos3y)(-sin3y)(3y') + 2xcos^2 3y - 3y'cos3y = 0$$
$$\Rightarrow 3y'[x^2(2cos3y)(-sin3y) - cos3y] = -2xcos^2 3y$$
$$\Rightarrow y' = \frac{2xcos^2 3y}{3[x^2(2cos3y)sin3y + cos3y]}$$

The slope of the tangent line m = $y'|_{(0,\frac{\pi}{3})} = \frac{2xcos^2 3y}{3[x^2(2cos3y)sin3y+cos3y]}|_{(0,\frac{\pi}{3})} = 0$

$\Rightarrow$ The tangent line is $y = \frac{\pi}{3}$

\# y'算對得 7 分, 剩下得 3 分