

微積分會考-考古題

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科目：微積分

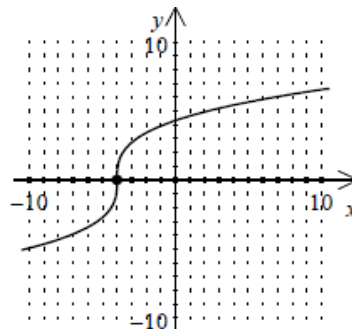
考試時間：100 分鐘

※ 注意：

- (一) 本試題共有單選題 20 題(選項：五選一)，填空題 10 題。
- (二) 選擇題請選出一個正確或最適當的答案，用 2B 鉛筆依題號於試卡上清楚劃記，複選作答者，該題不予計分。
- (三) 填空題請將正確答案依題號填於答案卷空格中。
- (四) 本試題不可使用電子計算器。
- (五) 本試題不得攜帶出考場。

一、選擇題：

1. Calculate $\lim_{x \rightarrow 4} \frac{\sqrt{x}-2}{x-4} =$
(A) ∞ (B) $-\infty$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) $\frac{1}{\sqrt{2}}$
2. Choose an equation from the following that expresses the fact that a function f is continuous at the number 6.
(A) $\lim_{x \rightarrow 0} f(x) = 6$ (B) $\lim_{x \rightarrow 0} f(x) = f(6)$ (C) $\lim_{x \rightarrow 6} f(x) = 6$
(D) $\lim_{x \rightarrow 6} f(x) = f(0)$ (E) $\lim_{x \rightarrow 6} f(x) = f(6)$
3. Let $f(x) = \begin{cases} x+7, & x < 3 \\ -x^2+9x-7, & x \geq 3 \end{cases}$. Which one of the following statements is TRUE?
(A) $\lim_{x \rightarrow 3^-} f(x) = 11$ (B) $\lim_{x \rightarrow 3^+} f(x) = 11$ (C) $\lim_{x \rightarrow 3} f(x) = 11$
(D) $f(3) = 10$ (E) f is continuous at 3
4. Determine where the function is differentiable.
(A) $x = -4$ (B) $y = 0$
(C) $y \neq 5$
(D) All x except $x = -4$
(E) The function is differentiable everywhere.



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5. If $\lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c} = a$, $a \in \mathbb{R}$, which one of the following statements is FALSE?
 (A) $f(c) = 0$ (B) $f'(c) = a$ (C) f is differentiable at c
 (D) f is continuous at c (E) $\lim_{x \rightarrow c} f(x) = f(c)$
6. Find the equation of the tangent line to the curve $f(x) = x^3 - x^2 + 6$ at the point $(1, 6)$.
 (A) $y = 2x + 4$ (B) $y = x + 5$ (C) $2y = x + 11$ (D) $y = -x + 7$
 (E) $y = -2x + 8$
7. Evaluate $\frac{d}{dx} \int_0^1 e^{x^2} dx =$
 (A) e^{x^2} (B) $2xe^{x^2}$ (C) $e - 1$ (D) $2e$ (E) 0
8. The annual sales x of a new consumer product can be modeled by $x = \frac{15,000t^2}{36 + t^2}$, where t is the number of years since the introduction of the product. Find the time at which the sales are increasing most rapidly.
 (A) $2\sqrt{3}$ years (B) $3\sqrt{2}$ years (C) $3\sqrt{3}$ years
 (D) $3\sqrt{5}$ years (E) $2\sqrt{6}$ years
9. Let $f(x) = x \ln x$, $x > 0$. Then the minimum value of $f(x)$ is:
 (A) 0 (B) 1 (C) e (D) $-e$ (E) $-\frac{1}{e}$
10. Evaluate $\int_2^5 [x] dx$, where $[x]$ is the largest integral less than or equal to x .
 (A) 7 (B) 8 (C) 9 (D) 10 (E) 10.5
11. The indefinite integral $\int x^2 \ln 5x dx$
 (A) $\frac{x^2}{2} (2 \ln 5x - 1) + c$ (B) $\frac{x^3}{3} (3 \ln 5x + 1) + c$ (C) $\frac{x^2}{4} (2 \ln 5x + 1) + c$
 (D) $\frac{x^3}{9} (3 \ln 5x - 1) + c$ (E) $\frac{x^2}{15} (2 \ln 5x - 1) + c$
12. The indefinite integral of $\int \frac{1}{x^2 - 1} dx$ is :
 (A) $\frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| + c$ (B) $\ln |x^2 - 1| + c$ (C) $\frac{1}{x} \ln |x^2 - 1| + c$
 (D) $\tan^{-1} x + c$ (E) $\sec^{-1} x + c$

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13. Find the particular solution, $y = f(x)$, that satisfies the conditions

$$f''(x) = \frac{6}{\sqrt{x}} + 3, \quad f'(1) = 12, \quad f(4) = 56.$$

- (A) $f(x) = 12x\sqrt{x} + \frac{3}{2}x^2 + 6x - 88$ (B) $f(x) = 3x\sqrt{x} + \frac{3}{2}x^2 + 7x - 20$
 (C) $f(x) = 8x\sqrt{x} + \frac{3}{2}x^2 - 3x - 20$ (D) $f(x) = 12\sqrt[3]{x} + \frac{3}{2}x^2 + 6x - 8$
 (E) $f(x) = 3\sqrt[3]{x} + \frac{3}{2}x^2 + 6x - 8$

14. The definite integral of $\int_{-1}^1 (e^{-x} + 1)dx$ is:

- (A) $-e^{-x} + x + c$ (B) $e - e^{-1} + 2$ (C) $e + e^{-1}$ (D) 2 (E) 0

15. Find the area of the region bounded by the graphs $f(y) = y^2 + 1$ and $g(y) = 4 - 2y$.

- (A) $\frac{31}{3}$ (B) $\frac{23}{4}$ (C) $\frac{31}{4}$ (D) $\frac{32}{3}$ (E) $\frac{22}{3}$

16. If f is continuous and $\int_0^4 f(x)dx = 10$, find $\int_0^2 f(2x)dx$.

- (A) 5 (B) 10 (C) 15 (D) 20 (E) 25

17. The indefinite integral of $\int e^{-(x-\theta)} dx$ is

- (A) $-\frac{1}{x-\theta}e^{-(x-\theta)} + c$ (B) $-\frac{1}{\theta}e^{-(x-\theta)} + c$ (C) $-\frac{1}{x}e^{-(x-\theta)} + c$
 (D) $-\theta e^{-(x-\theta)} + c$ (E) $-e^{-(x-\theta)} + c$

18. Find the limit of $\lim_{x \rightarrow \infty} (1 + \frac{2}{x})^{5x}$.

- (A) 1 (B) e^5 (C) e^{10} (D) ∞ (E) 0

19. Let $f(x, y) = y^3 - 3yx^2 - 3y^2 - 3x^2 + 1$. Which one of the following points is NOT a critical point of f ?

- (A) $(\sqrt{3}, -1)$ (B) $(-\sqrt{3}, 1)$ (C) $(-\sqrt{3}, -1)$ (D) $(0, 2)$ (E) $(0, 0)$

20. Evaluate the value of $\int_0^2 \int_0^x xy dy dx$?

- (A) $\frac{1}{2}$ (B) $\frac{4}{3}$ (C) 1 (D) 2 (E) 4

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二、填充題：

1. Let $f(x) = \frac{x^2 + x - 6}{|x - 2|}$, $x \neq 2$. Then $\lim_{x \rightarrow 2^-} f(x) = \underline{\hspace{2cm}}$
2. $\lim_{x \rightarrow \infty} (\sqrt{x^2 + x + 1} - \sqrt{x^2 - x + 1}) = \underline{\hspace{2cm}}$
3. Suppose that $f'(a) = 2$, then $\lim_{h \rightarrow 0} \frac{f(a + 2h) - (a - h)}{h} = \underline{\hspace{2cm}}$
4. $\lim_{x \rightarrow 0^+} x \ln x = \underline{\hspace{2cm}}$
5. Evaluate $\int \frac{x}{\sqrt{x^2 + 1}} dx = \underline{\hspace{2cm}}$
6. The improper integral of $\int_1^{\infty} \frac{\ln x}{x^2} dx = \underline{\hspace{2cm}}$
7. The area of the region bounded by the curves $y = e^x$ and $y = e^{-x}$ between $x = 0$ and $x = \ln 5$ is: $\underline{\hspace{2cm}}$
8. If $F(x) = \int_0^{\ln x} \sqrt{e^t + 1} dt$, then $F'(x) = \underline{\hspace{2cm}}$
9. Let $f(x, y) = y \ln x + 2xy^2$. The directional derivative of f at $(1, -2)$ is: $\underline{\hspace{2cm}}$
10. Evaluate the improper integral $\int_e^{\infty} \frac{1}{x(\ln x)^3} dx \underline{\hspace{2cm}}$.

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一、選擇題答案

1	2	3	4	5
C	E	B	D	A
6	7	8	9	10
B	E	A	E	C
11	12	13	14	15
D	A	C	B	D
16	17	18	19	20
A	E	C	B	D

二、填空題答案

1	2	3	4	5
-5	1	6	0	$\sqrt{x^2+1}+C$
6	7	8	9	10
1	$\frac{16}{5}$	$\frac{\sqrt{x+1}}{x}$	[6, -8]	$\frac{1}{2}$