

國立臺北大學 106 學年度第 1 學期 期中 考試命題紙

考試科目：微積分

開課班別：通訊 1/電機 1/智財學程

命題教授：吳漢銘

考試日期：11 月 06 日 (一) 10:00~11:30

※准帶項目打「○」，否則打「×」

1. 需加發計算紙或答案紙請在試題內封袋備註。

本試題共 1 頁，印刷份數：77 份

計算機	課本	筆記	電子辭典	紙本字典
-----	----	----	------	------

2. 為環保節能減碳，試題一律採雙面印刷，如有特殊印製需求，請註記：

備註：注意事項要看!!

×	×	×	×	×
---	---	---	---	---

注意事項：(1) 答案卷請寫上姓名及學號。(2) 請按題號順序書寫。(3) 每一題號需置於答案卷最左邊。(4) 可用鉛筆。(5) 需要計算過程。(6) 同時交回答案卷、題目卷、計算紙。(7) 總分共 120 分。

- (10 分) (a) What is the continuity of a function at a point? (b) State and prove that the Differentiability implies Continuity.
- (10 分) Can $f(x) = \frac{x(x^2 - 1)}{|x^2 - 1|}$ be extended to be continuous at $x = 1$ or -1 ? Give reasons for your answers.
- (20 分) Find the limits: (a) $\lim_{x \rightarrow 1} \frac{x^{50} - 1}{x - 1}$. (b) $\lim_{x \rightarrow \infty} \frac{x + \sin x + 2\sqrt{x}}{x + \sin x}$.
- (10 分) Find $\lim_{x \rightarrow -2} \frac{\lfloor 2x \rfloor + 2}{2x + 2}$, where $\lfloor \cdot \rfloor$ is the Gauss' symbol.
($y = \lfloor x \rfloor$: the function whose value at any number x is the greatest integer less than or equal to x .)
- (10 分) At what points is the function $y = |x - 1| + \sin x$ continuous?
- (10 分) Use the Chain Rule to find the value of $(f \circ g)'$ at the given value of x :
$$f(u) = \left(\frac{2u - 1}{2u + 1}\right)^2, u = g(x) = \frac{1}{x^2} - 1, x = -1.$$
- (20 分) Use implicit differentiation to find
(a) dy/dx if $x \cos(2x + 3y) = y \sin 5x$.
(b) d^2y/dx^2 if $y^2 = 1 - \frac{2}{x}$.
- (10 分) (a) Find the linearization of $f(x) = (1 + 2x)^k$ at $x = 0$. (b) Use the approximation in (a) to estimate $(1.0002)^{50}$.
- (20 分) Let $f(x)$ and $g(x)$ be differentiable functions. Use the definition of the derivative of a function to prove The Derivative Quotient Rule.

注意：1、考試求公平及公正，請同學務必自律，維護學校與學生之榮譽。

2、考試時不得有交談、窺視、夾帶、抄襲、傳遞、代考或其它作弊等舞弊行為，考畢務必交卷，不得攜卷出場，違者依考場規則議處。