THOMAS' CALCULUS (12/E)

14.2 Limits and Continuity in Higher Dimensions

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1 Limits for Functions of Two Variables

1.1 *Definition*

We say that a function ________ approaches the limit ______ as ________ approaches ________, and write

$$\underline{\hspace{6cm}}$$

if, for every number $\epsilon > 0$, there exists a corresponding number $\delta > 0$ such that for all (x, y) in the domain of f,

________ whenever ________.

1.2 *Theorem 1: Properties of Limits of Functions of Two Variables*

The following rules hold if L, M and k are real numbers and

$$\lim_{(x,y)\to(x_{)},y_0)} f(x,y) = \_\_\_\_\_\_ \quad \text{and} \quad \lim_{(x,y)\to(x_{)},y_0)} g(x,y) = \_\_\_\_\_\_$$

(a) *Sum Rule:* $\lim_{(x,y)\to(x_{)},y_0)} (f(x,y) + g(x,y)) =$ ________

(b) *Difference Rule:* $\lim_{(x,y)\to(x_{)},y_0)} (f(x,y) - g(x,y)) =$ ________

(c) *Constant Multiple Rule:* $\lim_{(x,y)\to(x_{)},y_0)} kf(x,y) =$ ________

(d) *Product Rule:* $\lim_{(x,y)\to(x_{)},y_0)} (f(x,y) \cdot g(x,y)) =$ ________

(e) *Quotient Rule:* $\displaystyle\lim_{(x,y)\to(x_{)},y_0)} \frac{f(x,y)}{g(x,y))} =$ ________, $M \neq 0.$

(f) *Power Rule:* $\displaystyle\lim_{(x,y)\to(x_{)},y_0)} [f(x,y)]^n =$ ________, $n \in N^+.$

(g) *Root Rule:* $\displaystyle\lim_{(x,y)\to(x_{)},y_0)} \sqrt[n]{f(x,y)} =$ ________, $n \in N^+.$

Ex. 1 .... (example1, p757)

(a) $\displaystyle\lim_{(x,y)\to(0,1)} \frac{x-xy+3}{x^2y+5xy-y^3} =$

(b) $\displaystyle\lim_{(x,y)\to(3,-4)} \sqrt{x^2+y^2} =$

Ex. 2 .... (example2, p757)

Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2-xy}{\sqrt{x}-\sqrt{y}}.$

sol:

2 Continuity

2.1 *Definition*

A function $f(x,y)$ is ______________ at the point (x_0,y_0) if

(a) f is ___________ at (x_0,y_0),

(b) ____________________ exists,

(c) ______________________________.

A function is ______________ if it is continuous at every point of its domain.

2.2 *Two-Path Test for Nonexistence of a Limit*

If a function $f(x, y)$ has a limits along ____________________ in the domain of f as (x, y) approaches (x_0, y_0), then $\lim_{(x,y)\to(x_0,y_0)} f(x, y)$ does not exist.

實習課練習 (EXERCISE 14.2)

2. $\displaystyle\lim_{(x,y)\to(0,4)} \frac{x}{\sqrt{y}}$

3. $\displaystyle\lim_{(x,y)\to(3,4)} \sqrt{x^2+y^2-1}$

7. $\displaystyle\lim_{(x,y)\to(0,\ln 2)} e^{x-y}$

12. $\displaystyle\lim_{(x,y)\to(\pi/2,0)} \frac{\cos y+1}{y-\sin x}$

14. $\displaystyle\lim_{\substack{(x,y)\to(1,1)\\ x\neq y}} \frac{x^2-y^2}{x-y}$

18. $\displaystyle\lim_{\substack{(x,y)\to(2,2)\\ x+y\neq 4}} \frac{x+y-4}{\sqrt{x+y}-2}$

21. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2}$

22. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{1-\cos xy}{xy}$