THOMAS' CALCULUS (12/E)

12.2 Vectors

開課班級: (105-2) 通訊1/電機1/智財學程 微積分
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1 Component Form

1.1 *Definition*

The vector represented by the _____ has initial point _____ and terminal point _____ and its length is denoted by _____. Two vectors are equal if they have the _____ and _____.

Definition

(a) If $\vec{v}$ is a two-dimensional vector in the plane equal to the vector with initial point at the _____ and terminal point _____, then the component form of _____ is _____.

(b) If $\vec{v}$ is a three-dimensional vector in the plane equal to the vector with initial point at the _____ and terminal point _____, then the component form of $\vec{v}$ is _____.

1.2 The magnitude or length of the vector $\vec{v} = \vec{PQ}$, $P(x_1, y_1, z_1), Q(x_2, y_2, z_2)$, is the nonnegative number

$$\|\vec{v}\| = \_\_\_\_\_ = \_\_\_\_\_$$

✎ Ex. 1 ..... (example1, p666)

Find the component form and length of the vector with initial point $P(-3, 4, 1)$ and terminal $Q(-5, 2, 2)$.

sol:

2 Vector Algebra Operations

2.1 *Definition*

Let $\vec{u} =< u_1, u_2, u_3 >$ and $\vec{v} =< v_1, v_2, v_3 >$ be vectors with k a scalar.

(a) Addition: $\vec{u} + \vec{v} =$ _____

(b) Scalar multiplication: $k\vec{u} =$ _____

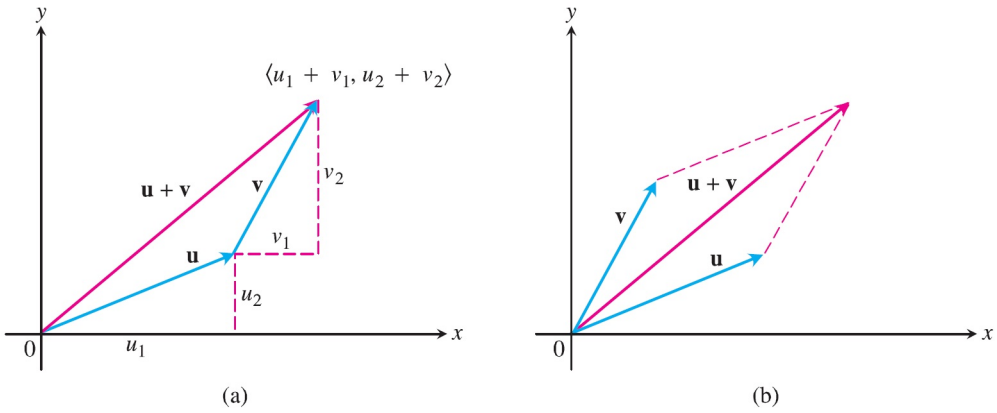

FIGURE 12.12 (a) Geometric interpretation of the vector sum. (b) The parallelogram law of vector addition.

2.2 *Properties of Vector Operations*

Let $\vec{u}, \vec{v}, \vec{w}$ be vectors and a, b be scalars.

1. $\vec{u} + \vec{v} =$ _____ 2. $(\vec{u} + \vec{v}) + \vec{w} =$ _____
3. $\vec{u} + \vec{0} =$ _____ 4. $\vec{u} + (-\vec{u}) =$ _____
5. $0\vec{u} = \vec{0}$ 6. $1\vec{u} = \vec{u}$
7. $a(b\vec{u}) =$ _____ 8. $a(\vec{u} + \vec{v}) =$ _____
9. $(a + b)\vec{u} =$ _____

2.3 A vector $\vec{v}$ of length 1 is called _____.

2.4 The standard unit vector are _____, _____, and _____.

2.5 Any vector $\vec{v} =< v_1, v_2, v_3 >$ can be written as a linear combination of the standard

unit:

$$\vec{v} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$
$$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$
$$= \_\_\_\_\_\_\_\_$$

2.6 The scalar ________ is the ________ (j-component, k-component) of the vector $\vec{v}$.

2.7 The vector from $P(x_1, y_1, z_1)$ to $Q(x2, y_2, z_2)$ is

$$\vec{PQ} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

2.8 Whenever $\vec{u} \neq \vec{0}$, ________ is a unit vector in the direction of $\vec{v}$.

2.9 The equation $\vec{v} =$ ________ expresses $\vec{v}$ as its length times its direction.

2.10 The midpoint M of the line segment joining points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is the point ________________.

Ex. **2** .... (`example3, p668`)

Let $\vec{u} =< -1, 3, 1 >$ and $\vec{v} =< 4, 7, 0 >$. Find the component of (a) $2\vec{u} + 3\vec{v}$ (b) $\vec{u} - \vec{v}$ (c) $\|\frac{1}{2}\vec{u}\|$.

sol:

Ex. **3** .... (`example4, p669`)

Find a unit vector $\vec{u}$ in the direction of the vector from $P_1(1, 0, 1)$ to $P_2(3, 2, 0)$.

sol:

實習課練習 (EXERCISE 12.2)

☐ In Exercise 17-22, express each vector in the form $\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$.

18. $\vec{P_1P_2}$ if $\vec{P_1}$ is the point $(1, 2, 0)$ and P_2 is the point $(-3, 0, 5)$.

22. $-2\vec{u} + 3\vec{v}$ if $\vec{u} =< -1, 0, 2 >$ and $\vec{v} =< 1, 1, 1 >$.

25. Express $2\vec{i} + \vec{j} - 2\vec{k}$ as a product of its length and direction.

33. Find a vector of magnitude 7 in the direction of $\vec{v} = 12\vec{i} - 5\vec{k}$.