

THOMAS' CALCULUS (12/E)

10.8 Taylor and Maclaurin Series

開課班級: (105-2) 通訊1/電機1/智財學程 微積分

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1 Series Representations

1.1 Assume that $f(x)$ is the sum of a power series

$$f(x) = \underline{\hspace{10em}}$$

$$= \underline{\hspace{10em}}$$

$$f'(x) = \underline{\hspace{10em}}$$

$$f''(x) = \underline{\hspace{10em}}$$

$$f'''(x) = \underline{\hspace{10em}}$$

⋮

$$f^{(n)}(x) = \underline{\hspace{10em}}$$

1.2 $f^{(n)}(a) = \underline{\hspace{2em}} \Rightarrow \underline{\hspace{4em}}$

1.3 $f(x) = \underline{\hspace{10em}}$


1.4 *Definitions: Taylor Series, Maclaurin Series*

Let f be a function with derivatives of all orders throughout some interval containing a as an interior point.

(a) Then the _____ generated by f at $x = a$ is _____

(a) The _____ generated by f is _____

_____ the Taylor series generated by f at $x = 0$.

 **Ex. 1** (example1, p585)

Find the Taylor series generated by $f(x) = 1/x$ at $a = 2$. Where, if anywhere, does the series converge to $1/x$?

sol:

2 Taylor Polynomials

2.1 The linearization of a differentiable function f at a point a is the polynomial of degree one given by


$$P_1(x) = \underline{\hspace{10em}}$$

We used this _____ to approximate $f(x)$ at values of x near a .

2.2 *Definitions: Taylor Polynomial of Order n*


Let f be a function with derivatives of order k for $k = 1, 2, \dots, N$ in some interval containing a as an interior point. Then for any integer n from 0 through N , the _____ generated by f at $x = a$ is the polynomial

$$P_n(x) = \underline{\hspace{10em}}$$

 **Ex. 2** (example2, p586)

Find the Taylor series and the Taylor polynomials generated by $f(x) = e^x$ at $x = 0$.

sol:

 **Ex. 3** (example3, p587)

Find the Taylor series and the Taylor polynomials generated by $f(x) = \cos x$ at $x = 0$.

sol:

實習課練習 (EXERCISE 10.8)

Find the Taylor polynomials of order 0,1,2, and 3 generated by f at a .

5. $f(x) = \frac{1}{x}$, $a = 2$

10. $f(x) = \sqrt{1-x}$, $a = 0$

Find the Maclaurin series.

12. xe^x

15. $\sin 3x$

Find the Taylor series generated by f at $x = a$.

23. $f(x) = x^3 - 2x + 4$, $a = 2$

27. $f(x) = 1/x^2$, $a = 1$

30. $f(x) = 2^x$, $a = 1$