THOMAS' CALCULUS (12/E)

10.7 Power Series

開課班級: (105-2) 通訊1/電機1/智財學程 微積分
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1 Power Series, The Convergence Theorem

1.1 *Definitions: Power Series, Center, Coefficients*

A power series about ________ is a series of the form

____

A power series about ________ is a series of the form

____

in which the center a and the coefficients $c_0, c_1, c_2, \cdots, c_n$ are constants.

2 The Radius of Convergence of a Power Series

2.1 *Theorem 18: The Convergence Theorem for Power Series*

If the power series $\sum_{n=0}^{\infty} a_n x^n$ converges for $x = c \neq 0$,then it converges absolutely for all x with __________. If the series diverges for $x = d$, then it diverges for all x with __________.

2.2 *COROLLARY TO THEOREM 18*

The convergence of the series $\sum c_n(x-a)^n$ is the one of the following three:

(a) There is a positive number R such that the series diverges for x with ____________ but converges absolutely for x with ____________. The series may or may not converge at either of the endpoints ____________ and ____________.

(b) The series converges absolutely for ________________.

(c) The series converges at ________ and diverges ________________.

2.3 R is called the ____________________ of the power series and the interval of radius R centered at $x = a$ is called the interval of convergence.

2.4 *How to Test a Power Series for Convergence*

(a) Use the ________ Test (or ___________ Test) to find the interval where the series converges ____________. Ordinarily, this is an open interval ____________ *or* ____________________

(b) If the interval of absolute convergence is ________, test for convergence or divergence at ______________. Use a _____________ Test, the __________ Test, or the __________________ Test.

(c) If the interval of absolute convergence is __________________, the series diverges for _____________ (it does not even converge conditionally), because the nth term does not approach zero for those values of x.

Ex. 1 .... (example1, p575)

Find the sum of a geometric power series with $a = 1$, $r = x$:

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \cdots + x^n + \cdots$$

sol:

✎ Ex. 2 .... (example2, p576)

Find the sum of a geometric power series with $a = 1$, $r = (x-2)/2$:
$1 - \frac{1}{2}(x-2) + \frac{1}{4}(x-2)^2 + \cdots + (-\frac{1}{2})^n(x-2)^n + \cdots$

sol:

✎ Ex. 3 .... (example3(a), p576)

For what values of x do the power series $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}$ converge?

sol:

✎ Ex. 4 .... (example3(b), p577)

For what values of x do the power series $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^{2n-1}}{2n-1}$ converge?

sol:

✎ Ex. **5** .... (`example3(c), p577`)

For what values of x do the power series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ converge?

sol:

✎ Ex. **6** .... (`example3(d), p577`)

For what values of x do the power series $\sum_{n=0}^{\infty} n!x^n$ converge?

sol:

實習課練習 (EXERCISE 10.7)

□ Find the series's radius and interval of convergence. Foe what values of x does the series converge absolutely, or conditionally?

3. $\displaystyle\sum_{n=0}^{\infty}(-1)^n(4x+1)^n$

8. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n(x+2)^n}{n}$

11. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{n!}$

15. $\displaystyle\sum_{n=0}^{\infty}\frac{x^n}{\sqrt{n^2+3}}$

23. $\displaystyle\sum_{n=1}^{\infty}(1+\frac{1}{n})^n x^n$

34. $\displaystyle\sum_{n=1}^{\infty}\frac{3\cdot 5\cdot 7\cdots(2n+1)}{n^2\cdot 2^n}x^{n+1}$

36. $\displaystyle\sum_{n=0}^{\infty}(\sqrt{n+1}-\sqrt{n})(x-3)^n$