THOMAS' CALCULUS (12/E)

10.6 Alternating Series, Absolute and Conditional Convergence

開課班級: (105-2) 通訊1/電機1/智財學程 微積分
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1 Alternating Series

1.1 A series in which the terms are alternately _____ and _____ is an _____.

(a) alternating harmonic series: _____

(b) geometric series with $r = -1/2$: _____

(c) $1 - 2 + 4 - 4 + 5 - 6 + \cdots + (-1)^{n+1}n + \cdots$

1.2 *Theorem 14: The Alternating Series Test (Leibniz's Theorem)*

The series

$$\sum_{n=1}^{\infty}(-1)^{n+1}u_n = u_1 - u_2 + u_3 - u_4 + \cdots$$

converges if all three of the following conditions are satisfied:

(a) _____ are all _____.

(b) _____ for all $n \geq N$, for some integer N.

(c) _____.

✎ Ex. 1 ..... (example1, p569)

Does the alternating harmonic series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n} = 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$ converge?

sol:

2 Absolute and Conditional Convergence

2.1 *Definitions: Absolutely Convergent*

A series _____ (is _____) if the corresponding series of absolute values, _____

2.2 *Definitions: Conditionally Convergent*

A series that converges but does not converge absolutely converges _____.

2.3 The geometric series $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\cdots$ converges _____ because _____ converges.

2.4 The alternating hramonic series does not converge absolutely. It converges _____.

2.5 *Theorem 16: The Absolute Convergence Test*

If _____ converges, then _____ converges.

✎ Ex. 2 ..... (example4, p571)

(a) Does $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^2}$ converge absolutely?

(b) Does $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ converge absolutely?

✎ Ex. **3** ..... (example5, p571)

Determine the convergence (absolutely convergence, conditionally convergence) of the alternating p-series: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p}$.

sol:

3 Summary of Tests for Convergence and Divergence of Series

3.1 The nth-Term Test: _____.

3.2 Geometric series: _____.

3.3 p-series: _____.

3.4 Series with nonnegative terms:
Try _____.
Try _____.

3.5 Series with some negative terms: _____.

3.6 Alternating series: _____.

實習課練習 (EXERCISE 10.6)

☐ Determining convergence or divergence.

2. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^{3/2}}$

11. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\ln n}{n}$

13. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sqrt{n}+1}{n+1}$

☐ Which of the series converge absolutely, which converge and which diverge?

16. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(0.1)^n}{n}$

20. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n!}{2^n}$

25. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1+n}{n^2}$

30. $\sum_{n=2}^{\infty}(-1)^{n}\frac{\ln n}{n-\ln n}$

39. $\sum_{n=1}^{\infty}(-1)^{n}\frac{(2n)!}{2^n n! n}$

42. $\sum_{n=1}^{\infty}(-1)^{n}(\sqrt{n^2+n}-n)$

44. $\sum_{n=1}^{\infty}(-1)^{n}\frac{1}{\sqrt{n}+\sqrt{n+1}}$