

## THOMAS' CALCULUS (12/E)

**10.5 The Ratio and Root Tests**

開課班級: (105-2) 通訊1/電機1/智財學程 微積分

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# 1 The Ratio Test

## 1.1 Theorem 12: The Ratio Test


Let  $\sum a_n$  be a series with positive terms and suppose that

Then \_\_\_\_\_

(a) the series converges if \_\_\_\_\_,

(b) the series diverges if \_\_\_\_\_ or \_\_\_\_\_,

(c) the test is inconclusive if \_\_\_\_\_.

 **Ex. 1** ..... (example1, p564)

Investigate the convergence of the series.

(a) 
$$\sum_{n=0}^{\infty} \frac{2^n + 5}{3^n}$$

(b) 
$$\sum_{n=1}^{\infty} \frac{(2n)!}{n!n!}$$

$$(c) \sum_{n=1}^{\infty} \frac{4^n n! n!}{(2n)!}$$

## 2 The Root Test

### 2.1 Theorem 13: The Root Test

Let  $\sum a_n$  be a series with  $a_n \geq 0$  and suppose that \_\_\_\_\_ Then

- (a) the series converges if \_\_\_\_\_,
- (b) the series diverges if \_\_\_\_\_ or \_\_\_\_\_,
- (c) the test is inconclusive if \_\_\_\_\_.

 **Ex. 2** ..... (example2, p566)

Investigate the convergence of the series.

$$(a) \sum_{n=1}^{\infty} \frac{n^2}{2^n}$$

$$(b) \sum_{n=1}^{\infty} \frac{2^n}{n^2}$$

$$(c) \sum_{n=1}^{\infty} \left(\frac{1}{1+n}\right)^n$$

## 實習課練習 (EXERCISE 10.5)

Use the Ratio Test to determine if each series converges or diverges.

3.  $\sum_{n=1}^{\infty} \frac{(n-1)!}{(n+1)^2}$ .

6.  $\sum_{n=2}^{\infty} \frac{3^{n+2}}{\ln n}$ .

Use the Root Test to determine if each series converges or diverges.

12.  $\sum_{n=1}^{\infty} \left( \ln \left( e^2 + \frac{1}{n} \right) \right)^{n+1}$ .

13.  $\sum_{n=1}^{\infty} \frac{8}{(3 + (1/n))^{2n}}$ .

Determining convergence or divergence.

18.  $\sum_{n=1}^{\infty} n^2 e^{-n}$

20.  $\sum_{n=1}^{\infty} \frac{n!}{10^n}$

23.  $\sum_{n=1}^{\infty} \frac{2 + (-1)^n}{1.25^n}$

27.  $\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$

32.  $\sum_{n=1}^{\infty} \frac{n \ln n}{2^n}$

36.  $\sum_{n=1}^{\infty} \frac{n 2^n (n+1)!}{3^n n!}$

38.  $\sum_{n=1}^{\infty} \frac{n!}{n^n}$