THOMAS' CALCULUS (12/E)

8.7 Improper Integrals

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1 Infinite Limits of Integration

1.1 Definitions: Type I Improper Integrals

Integrals with _____ of integration are **improper integrals of** Type I.

(a) If f(x) is continuous on $[a, \infty)$, then

$$\int_{a}^{\infty} f(x) \ dx = \underline{\hspace{1cm}}$$

(b) If f(x) is continuous on $(-\infty, b]$, then

$$\int_{-\infty}^{b} f(x) \ dx =$$

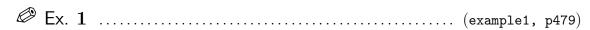
(c) If f(x) is continuous on $(-\infty, \infty)$ then

$$\int_{-\infty}^{\infty} f(x) \ dx =$$

where c is any real number. In each case, if the limit is _____ we say that the improper integral ____ and that the _____ is the **value** of the improper integral. If the limit fails to exist, the improper integral ____ (the area under the curve is infinite).

- 1.2 Examples:
 - (a) Upper limit: $\int_{1}^{\infty} \frac{\ln x}{x^2} dx =$
 - (b) Lower limit: $\int_{-\infty}^{0} \frac{dx}{1+x^2} =$

(c) Both limits: $\int_{-\infty}^{\infty}$	$\int_{0}^{\infty} \frac{dx}{1+x^2} =$
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Is the area under the curve $y=(\ln x)/x^2$ from x=1 to $x=\infty$ finite? If so, what is it?

sol:

Evaluate $\int_{\infty}^{\infty} \frac{dx}{1+x^2}$.

sol:

For what values of p does the integral $\int_1^\infty \frac{1}{x^p} dx$ converge? When the integral does converge, what is its value?

sol:

2 Integrands with Vertical Asymptotes

2.1 Definitions: Type II Improper Integrals

Integrals of functions that become _____ within the interval of integration are improper integrals of Type II.

(a) If f(x) is continuous on (a, b] and is discontinuous at a then

$$\int_{a}^{b} f(x) \ dx = \underline{\qquad}$$

(b) If f(x) is continuous on [a, b) and is discontinuous at b, then

$$\int_{a}^{b} f(x) \ dx = \underline{\qquad}$$

(c) If f(x) is discontinuous at c, where a < c < b, and continuous on $[a, c) \cup (c, b]$ then

$$\int_{a}^{b} f(x) \ dx =$$

In each case, if the limit is finite we say the improper integral ____ and that the limit is the value of the improper integral. If the limit does not exist, the integral ____ .

2.2 Examples:

(a) Upper endpoint: $\int_0^1 \frac{dx}{(x-1)^{2/3}} = \underline{\hspace{1cm}}$

(b) Lower endpoint: $\int_{1}^{3} \frac{dx}{(x-1)^{2/3}} = \frac{1}{(x-1)^{2/3}}$

(c) Interior point: $\int_0^3 \frac{dx}{(x-1)^{2/3}} = \underline{\hspace{1cm}}$

Ex. 4 (example4, p482)

Investigate the convergence of $\int_0^1 \frac{1}{1-x} dx$.

sol:

	Ex.	5				 	 	 	 	 (example5,	p482
Eva	luate		$\int_0^3 \frac{1}{(x)^2}$	$\frac{dx}{(-1)}$	${ 2/3 }$.						

sol:

3 Tests for Convergence and Divergence

3.1 Theorem 1: Direct Comparison Test

Let f and g be continuous on $[a, \infty)$ with _____ for all $x \ge a$. Then

(a)
$$\int_a^\infty f(x) dx$$
 ______ if $\int_a^\infty g(x) dx$ ______.

(b)
$$\int_a^\infty g(x) \ dx$$
 ______ if $\int_a^\infty f(x) \ dx$ ______.

 $3.2 \ \underline{\textit{Theorem 2: Limit Comparison Test}}$

Ex. 6 (example 6, p483)

Does the integral $\int_{1}^{\infty} e^{-x^2} dx$ converges?

sol:



- (a) $\int_1^\infty \frac{\sin^2 x}{x^2} dx$ converges because
- (b) $\int_{1}^{\infty} \frac{1}{\sqrt{x^2 0.1}} dx$ diverges because

實習課練習 (EXERCISE 8.7)

- $\hfill\Box$ Evaluate the integrals.
- 2. $\int_{1}^{\infty} \frac{dx}{x^{1.001}}$
- 7. $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$
- 10. $\int_{-\infty}^{2} \frac{2 dx}{x^2 + 4}$
- 13. $\int_{-\infty}^{\infty} \frac{2x \ dx}{(x^2+1)^2}$
- 17. $\int_0^\infty \frac{dx}{(1+x)\sqrt{x}}$
- **22.** $\int_0^\infty 2e^{-\theta}\sin\theta \ d\theta$
- **25.** $\int_0^1 x \ln x \ dx$
- **30.** $\int_{2}^{4} \frac{dt}{t\sqrt{t^2-4}}$
- **32.** $\int_0^2 \frac{dx}{\sqrt{|x-1|}}$
 - \square Test the integrals for convergence.
- **39.** $\int_0^{\ln x} x^{-2} e^{-1/x} dx$
- **45.** $\int_{-1}^{1} \ln|x| \ dx$
- **54.** $\int_{2}^{\infty} \frac{x \ dx}{\sqrt{x^4 1}}$
- $58. \int_{2}^{\infty} \frac{1}{\ln x} \ dx$
- $61. \int_1^\infty \frac{1}{\sqrt{e^x x}}$