

THOMAS' CALCULUS (12/E)

8.3 Trigonometric Substitutions

開課班級: (105-2) 通訊1/電機1/智財學程 微積分

授課教師: 吳漢銘 (國立臺北大學統計學系 副教授)

教學網站: <http://www.hmwu.idv.tw>

系級: _____ 學號: _____ 姓名: _____

1 Three Basic Substitutions

1.1 Integrals involving $\sqrt{a^2 - x^2}$:

Let $x =$ _____,

$$a^2 - x^2 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

1.2 Integrals involving $\sqrt{a^2 + x^2}$:

Let $x =$ _____,

$$a^2 + x^2 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

1.3 Integrals involving $\sqrt{x^2 - a^2}$:

Let $x =$ _____,

$$x^2 - a^2 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

1.4 (a) $\sqrt{a^2 + x^2} =$ _____

(b) $\sqrt{a^2 - x^2} =$ _____

(c) $\sqrt{x^2 - a^2} =$ _____

圖示如下:

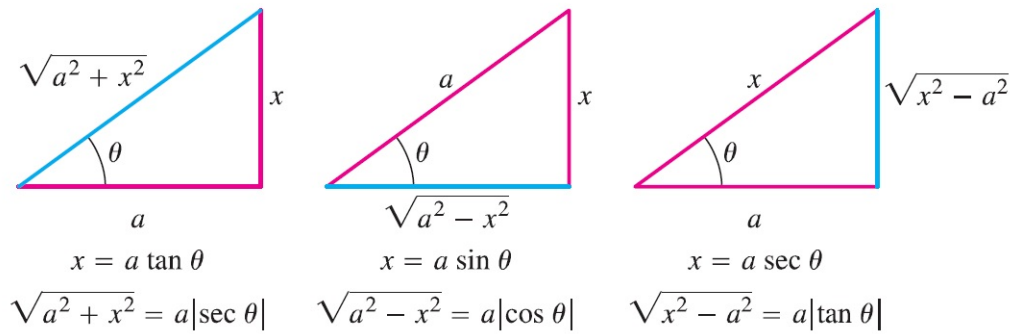




FIGURE 8.2 Reference triangles for the three basic substitutions identifying the sides labeled x and a for each substitution.

 **Ex. 1** (example1, p450)


Evaluate $\int \frac{dx}{\sqrt{4 + x^2}}$.

sol:

 **Ex. 2** (example2, p451)

Evaluate $\int \frac{x^2 dx}{\sqrt{9 - x^2}}$

sol:

 **Ex. 3** (example3, p451)

Evaluate $\int \frac{dx}{\sqrt{25x^2 - 4}}$, $x > 2/5$.

sol:

實習課練習 (EXERCISE 8.3)

4.
$$\int_0^2 \frac{dx}{8 + 2x^2}$$

7.
$$\int \sqrt{25 - t^2} dt$$

10.
$$\int \frac{5 dx}{\sqrt{25x^2 - 9}}, \quad x > 3/5$$

17.
$$\int \frac{x^3 dx}{\sqrt{x^2 + 4}}$$

20.
$$\int \frac{\sqrt{9 - w^2}}{w^2} dw$$

33.
$$\int \frac{v^2}{(1 - v^2)^{5/2}} dv$$

43.
$$\int \frac{x dx}{\sqrt{1 + x^4}}$$

46.
$$\int \sqrt{\frac{x}{1 - x^3}} dx$$