微積分會考

選擇題

1. As a rumor spreads across a college campus, the number of people that have heard it can be modeled by the equation, $N(t) = \frac{6000t^2+2700t}{(2t+3)^2}$, where t is days since the rumor started spreading. What happens to the number of people that have heard the rumor in the long run (as $t \to \infty$)?

   (a) 5000

   (b) 6000

   (c) 1500

   (d) 400

   Ans: c

2. During a medical procedure, the size of a roughly spherical tumor is estimated by measuring its diameter and using the formula $V = \frac{4}{3}\pi R^3$ (R: radius) to compute its volume. If the diameter is measured as 2.5 cm with a maximum error of 2%, what is the range of the volume measurement?

   (a) $6.85 \leq V \leq 9.27$

   (b) $7.69 \leq V \leq 8.67$

   (c) $4.78 \leq V \leq 7.53$

   (d) $5.23 \leq V \leq 6.12$

   Ans: b

3. Suppose that $F(x) = f(g(x))$ and $g(14) = 2$, $g'(14) = 5$, $f'(14) = 15$, and $f'(2) = 16$. Find $F'(14)$.

   (a) 20

   (b) 80

   (c) 17

   (d) 24

   Ans: b

4. Find the derivative of the function $f(x) = x^5$. State the domain of the function and the domain of its derivative.

   (a) $f'(x) = -5x^4$, $\mathbb{R}$, $\mathbb{R}$
   (b) $f'(x) = x^4$, $\mathbb{R}$, $\mathbb{R}^+$
   (c) $f'(x) = 5x^4$, $\mathbb{R}$, $\mathbb{R}$
   (d) $f'(x) = 5x^4$, $\mathbb{R}$, $\mathbb{R}^+$

   Ans: c

5. Consider a function

$$f(x) = \begin{cases} 2 - x, & \text{if } x < 0 \\ 2, & \text{if } x = 0 \\ \sqrt{4 + x^2}, & \text{if } x > 0 \end{cases}$$

   Which of the following is correct?

   (a) The function is continuous but not differentiable at $x = 0$
   (b) The function is differentiable but discontinuous at $x = 0$
   (c) None of above
   (d) The function is continuous and differentiable at $x = 0$

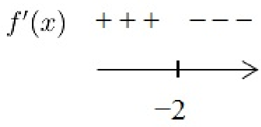
   Ans: a

6. A bus company will charter a bus that holds 52 people to groups of 34 or more. If a group contains exactly 34 people, each person pays \$65. In large groups, everybody's fare is reduced by \$1 for each person in excess of 34. Determine the size of the group for which the bus company's revenue will be greatest.

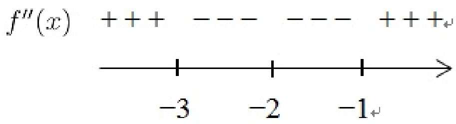
   (a) 34 with revenue \$2210.
   (b) Groups of 49 or 50 with revenue \$2450.
   (c) Groups of 49 or 50 with revenue \$2210.
   (d) 34 with revenue \$2450.

   Ans: b

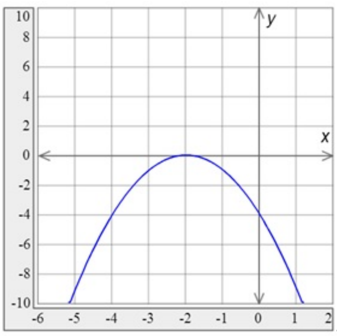
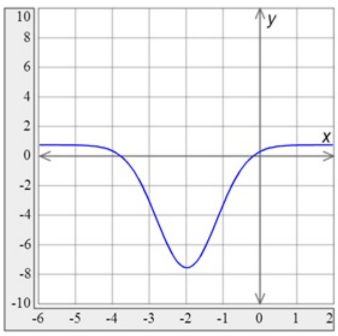
7. Diagrams indicating intervals of increase or decrease and concavity are given. Select a possible graph for a function with these characteristics.

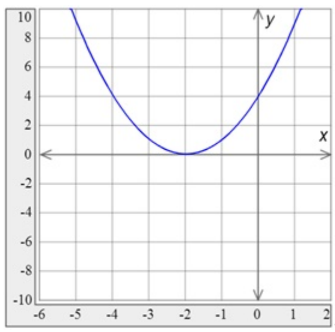
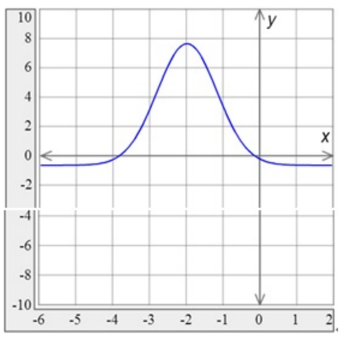


1: Sign of $f'(x)$



2: Sign of $f''(x)$

[a]  [b] 

[c]  [d] 

Ans: d

8. Differentiate the function $f(x) = \frac{\sqrt[3]{x+9}}{(3-8x)^9}$.

   (a) $f'(x) = \frac{\sqrt[3]{x+9}}{(3-8x)^9} \cdot [\frac{1}{3}\frac{1}{x+9} - \frac{72}{3-8x}]$

   (b) $f'(x) = \frac{\sqrt[3]{x+9}}{(3-8x)^9} \cdot [\frac{1}{3}\frac{1}{x+9} + \frac{72}{3-8x}]$

   (c) $f'(x) = \frac{x+9}{(3-8x)^9} \cdot [\frac{1}{3}\frac{1}{x+9} + \frac{72}{3-8x}]$

   (d) $f'(x) = \frac{1}{3}\frac{1}{x+9} + \frac{72}{3-8x}$

   Ans: b

9. Find the inflection point of the function $f(x) = xe^{-2x}$.

   (a) $(2, e^2)$

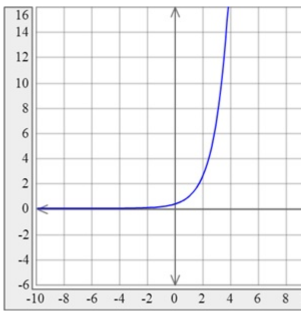
   (b) $(1, e^{-2})$

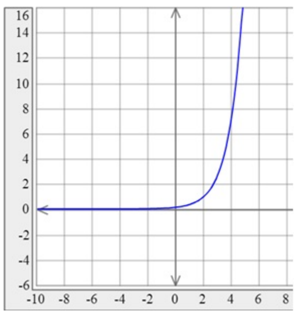
   (c) $(3, e^2)$

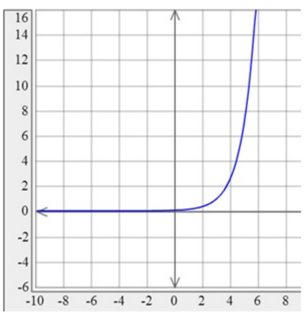
   (d) $(1, e^3)$

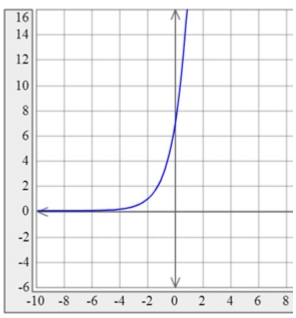
   Ans: b

10. Choose the correct graph for the given function $y = e^{x-2}$.

[a] 

[b] 

[c] 

[d] 

Ans: b

11. Find the relative extrema of the function $f(x) = \frac{1}{\sqrt{3\pi}}e^{-x^2/2}$. Round your answer to one decimal place.

    (a) Relative minimum is $(0, 0.3)$.
    (b) Relative minimum is $(0, -0.3)$; relative maximum is $(1, 0.3)$.
    (c) Relative minimum is $(-1, -0.3)$; relative maximum is $(1, 0.3)$.
    (d) Relative maximum is $(0, 0.3)$.

    Ans: d

12. Find the limit of $\lim_{x\to\infty}(1 + \frac{2}{x})^{5x}$

    (a) 1
    (b) e^{10}
    (c) e^3
    (d) ∞

    Ans: b

13. Given $f'(x) = \frac{x+1}{\sqrt{x}}$ and $f(4) = 5$. Find the function f.

    (a) $f(x) = \frac{2}{3}x^{3/2} + 2\sqrt{x} - 13/3$
    (b) $f(x) = x^3 - 2x^2 + 8x - 4$
    (c) $f(x) = x^{3/2}/3 + 2x^2 - 13/3$
    (d) $f(x) = 2x^3 - 2x - 13/3$

    Ans: a

14. Find $\int \frac{6e^x+6e^{-x}}{e^x-e^{-x}}dx$.

    (a) $6\ ln|e^x - e^{-x}| + C$.
    (b) $\frac{1}{6}\ ln|e^x + e^{-x}| + C$.
    (c) $6\ ln|e^x + e^{-x}| + C$.
    (d) $\frac{1}{6}\ ln|e^x - e^{-x}| + C$.

    Ans: a

15. Find $\int \frac{x}{x-7} dx$.

(a) $\int \frac{x}{x-7} dx = x + 7\ln|x-7| + C$

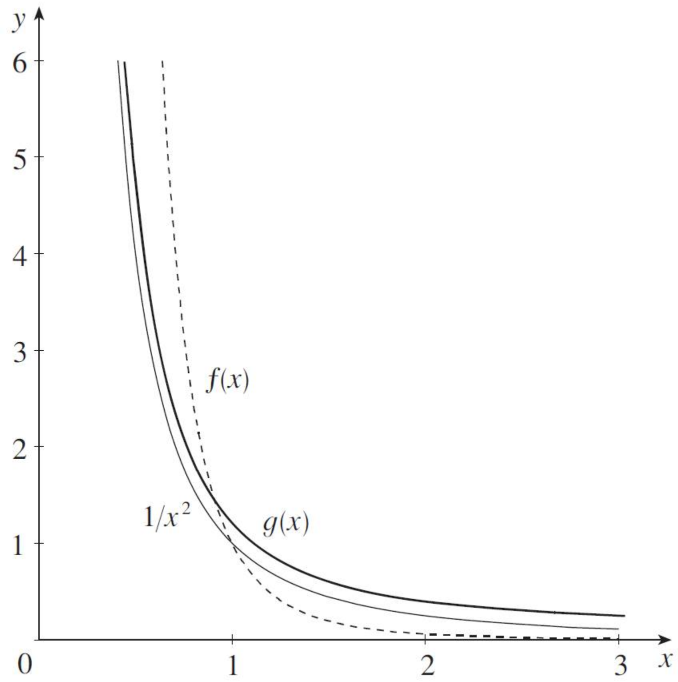
(b) $\int \frac{x}{x-7} dx = x + \ln|x-7| + C$

(c) $\int \frac{x}{x-7} dx = x - \ln|x-7| + C$

(d) $\int \frac{x}{x-7} dx = x - 7\ln|x-7| + C$

Ans: a

16. Consider the three functions in the following figure.



Which of the following is correct?

(a) $\int_0^1 \frac{1}{x^2} dx$ is convergent to 1.

(b) $\int_1^\infty \frac{1}{x^2} dx$ is convergent to 1.

(c) By using Comparison Theorem, $\int_0^1 g(x)dx$ is convergent.

(d) By using Comparison Theorem, $\int_1^\infty f(x)dx$ is divergent.

Ans: b

17. Find the relative minimum of the function, $f(x,y) = 2x^2 + y^2$, subject to the constraint $g(x,y) = x + y - 1 = 0$.

    (a) 2/3

    (b) 2.5

    (c) 3/5

    (d) 3

    Ans: a

18. Find the volume of the solid bounded above by the surface $z = f(x,y)$ and below by the plane region R, where $f(x,y) = 2x^2y$; R is the region bounded by the graphs of $y = x$ and $y = x^2$.

    (a) 2/35

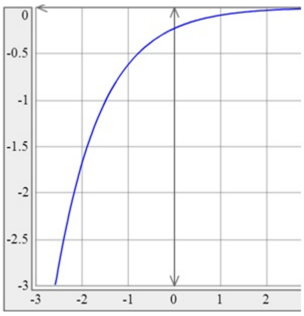
    (b) 13/5

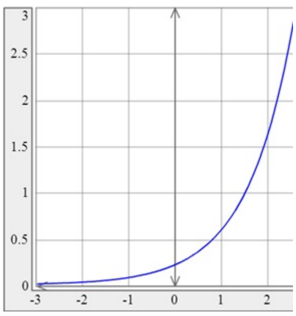
    (c) 17/2

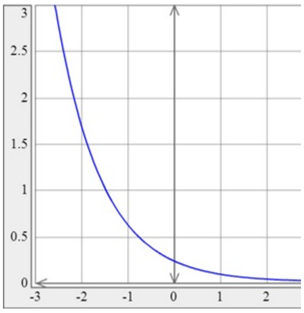
    (d) 4/7

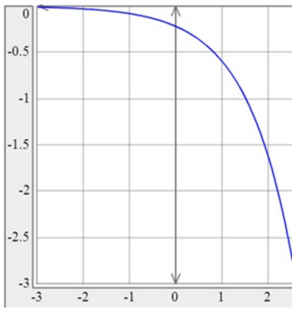
    Ans: a

19. Choose the correct sketch of the indicated level curve $f(x,y) = C$ for the given constant C. $f(x,y) = 9ye^x$; $C = 2$.

[a] 

[b] 

[c] 

[d] 

Ans: c

20. Describe the domain of the given function

$$f(x,y) = \frac{3x}{\ln(x+2y).}$$

(a) All ordered pairs (x, y) of real numbers for which $x + 2y < 0$ and $x + 2y = 1$.

(b) All ordered pairs (x, y) of real numbers for which $x + 2y > 0$ and $x + 2y \neq 1$.

(c) All ordered pairs (x, y) of real numbers for which $x + 2y < 0$ and $x + 2y \neq 1$.

(d) All ordered pairs (x, y) of real numbers for which $x + 2y > 0$ and $x + 2y = 1$.

Ans: b

填空題

1. Find the absolute maximum value and the absolute minimum value, if any, of the function, $f(x) = 3x^{2/3} - 2x$ on $[0, 3]$.

   Ans: the absolute maximum value: 1; the absolute minimum value: 0

2. Differentiate the given function. Give your answer in terms of natural logs with the arguments in parentheses [e.g. $\ln(x)$].

$$f(x) = \frac{\log_8 x}{14\sqrt{x}}$$

   Ans: $\frac{2-lnx}{28 \cdot ln8 \cdot x\sqrt{x}}$.

3. Find the area of the region bounded by the graphs of the functions $y = x^4 + 1$ and $y = 2x^2$.
Ans: $\frac{16}{15}$

4. Find the area of the region R that lies under the given curve $y = f(x)$ over the indicated interval $a \leq x \leq b$. Under $y = \frac{4}{x}$, over $1 \leq x \leq e^6$.
Ans: $\int_1^{e^6} \frac{4}{x} dx = 24$.

5. Compute $\int x^2 \ln(2x) dx$.

   Ans: $\frac{x^3}{3}(lnx + ln2 - \frac{1}{3}) + C$

6. Find the area of the region R that is completely enclosed by the graphs of the functions $f(x) = 4x$ and $g(x) = x^3 + 3x^2$.
Ans: Area $= 32$

7. The concentration of a drug t hours after being injected into a patient's bloodstream is $C(t) = 360te^{-\frac{t}{2}}$ mg/mL. What is the average concentration of drug in the patient's bloodstream over the first 12 hours after the injection?
Ans: $120 - \frac{840}{e^6}$

8. At a certain factory, the daily output is $Q(K, L) = 50K^{1/2}L^{1/3}$ units, where K denotes the capital investment measured in units of $\$1,000$ and L the size of the labor force measured in worker-hours. Suppose that the current capital investment is $\$625,000$ and that $2,197$ worker-hours of labor are used each day. Use marginal analysis to estimate the effect of an additional capital investment of $\$1,000$ on the daily output if the size of the labor force is not changed.
Ans: Daily output will increase by 13 units.

9. Evaluate the integral $\int_0^1 \int_{x^2}^1 xe^{y^2} dydx$.
Ans: $\frac{1}{4}(e - 1)$

10. Find the volume of the solid lying under the plane $z = 4 + x^2 - y^2$ and above the square $R = [-1, 1] \times [0, 2]$.
Ans: $\int_0^2 \int_{-1}^1 4 + x^2 - y^2 dxdy = 12$.